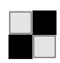
Fractal Designs for Quilts, Mosaics, and Other Decorated Surfaces, Part 1

The notion of "fractal art" may conjure up visions of quasi-organic curves and elaborate shapes, but the world of fractals also contains many designs consisting of squares or triangles. These designs are suitable for incorporating into quilts, mosaics, counted-thread embroideries, or any technique that decorates a grid or mesh.

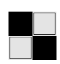
The designs I will explore in this article are all based on the Morse-Thue sequence, which is a mathematical concept you can enjoy without a background in the subject. (For formal details about about Morse-Thue, see [1], [2],) I was introduced to the visual possibilities of Morse-Thue in an article by Ralph Griswold [3].

The starting point is a list of rules saying how to transform a square pattern into a different square pattern. Each rule shows a "before" square and an "after" square.

The Light Rule tells us what to do when "before" is a light square.

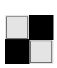
Before: , After: .

The Dark Rule tells us what to do when "before" is a dark square.

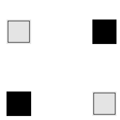
Before: , After: .

Now we'll apply the rules to build two pattern families, each one generated from a single square. First we'll start with a light square, and call it Generation 1:

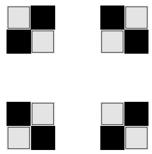
apply the Light Rule to Generation 1 to get Generation 2:



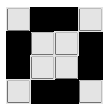
Now what should we do to create Generation 3? If we spread the squares in Generation 2 apart, keeping them in the same relative positions:



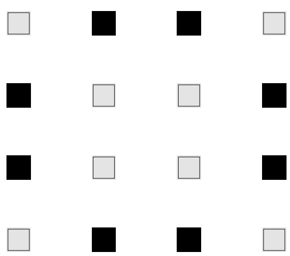
and apply the Light Rule to the light squares, and the Dark Rule to the dark squares, then we get this intermediate pattern:



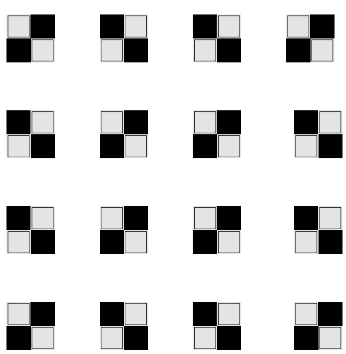
If we push the squares in the intermediate pattern together, we get Generation 3:



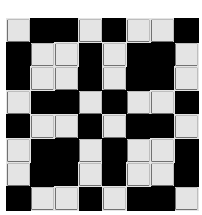
Let's do it again. We spread out the Generation 3 pattern into sixteen squares:



applying the Light Rule to the light squares, and the Dark Rule to the dark squares:



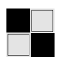
and push them all back together again to get Generation 4:



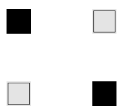
What would have happened differently if Generation 1 had been a dark square instead of a light square?

We've used the term "Generation" to refer to the patterns generated from the light square, so to avoid confusion, let's use the term D-Generation to label the patterns generated from the dark square. Here's D-Generation 1:

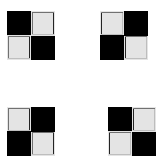
Apply the Dark Rule to this "before" pattern to get D-Generation 2:



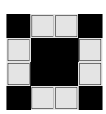
Spread the squares apart:



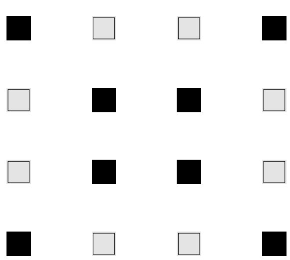
and apply the Light Rule to the light squares, and the Dark Rule to the dark squares:



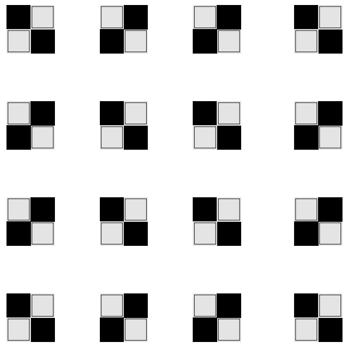
Push these together to get D-Generation 3:



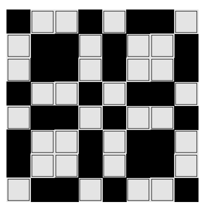
Of course, there is nothing to stop us from spreading out D-Generation 3:



applying the Light Rule to the light squares, and the Dark Rule to the dark squares:



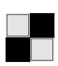
and pushing the squares together to get D-Generation 4:

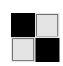


Before building any more patterns, let’s examine some relationships of the patterns we’ve built already by looking at them side-by-side.

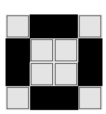
Generation 1 and D-Generation 1:

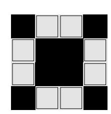
Generation 2 and D-Generation 2:



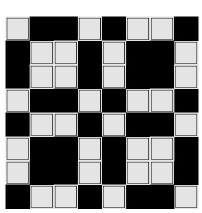


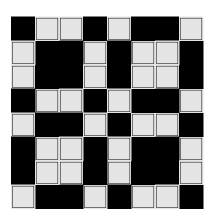
Generation 3 and D-Generation 3:





Generation 4 and D-Generation 4:





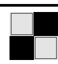
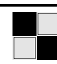
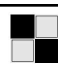
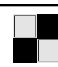
One relationship is that for every pattern number, the Dark and Light patterns are negatives of each other, that is, if you start with any pattern in the Light family and change all the dark squares to light, and all the light squares to dark, you will produce the pattern for the same generation from the Dark family.

Another relationship is a "shorthand" way to describe each pattern. Because we're calling the patterns in the Dark family the D-Generations, to be consistent, we really ought to call the patterns in the Light family the L-Generations.

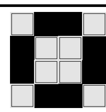
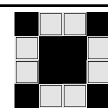
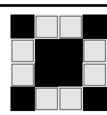
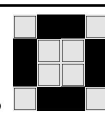
We can describe L-Generation 2 as made up from:

L-Generation 1 	D-Generation 1 
D-Generation 1 	L-Generation 1 

and L-Generation 3 as:

L-Generation 2 	D-Generation 2 
D-Generation 2 	L-Generation 2 

and L-Generation 4 as:

L-Generation 3 	D-Generation 3 
D-Generation 3 	L-Generation 3 

We could draw similar boxes to describe D-Generation 2 using L-Generation 1 and D-Generation 1, D-Generation 3 using D-Generation 2 and L-Generation 2, and so on.

These relationships are a consequence of the rules we have been following at each step.

The actions of spreading the squares and applying the rules to get

L-Generation 4,

required that we first performed the actions to get

L-Generation 3 and D-Generation 3,

which in turn required that we performed the actions to get

L-Generation 2 and D-Generation 2 from

L-Generation 1 and D-Generation 1.

These are a lot of words, and even more words are needed if we want to describe Generations 6, 7, 8, and so on. But the consequences should be clear: we no longer need to spread the pattern out into single squares to go to the next pattern, we can build L-Generation 4 and D-Generation 4 from the L-Generation 3 and D-Generation 3, the "5" generations from the "4" generations, the "6" generations from the "5" generations, and so on.

Somebody in the world of computing might write it this way:

To generate the Bi-Colored Morse-Thue Patterns in the plane,

let L-Generation 1 = □, D-Generation 1 = ■

Then for N greater than 1, let L-Generation N+1 =

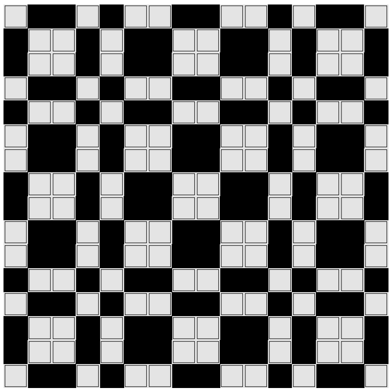
L-Generation N	D-Generation N
D-Generation N	L-Generation N

and D-Generation N+1 =

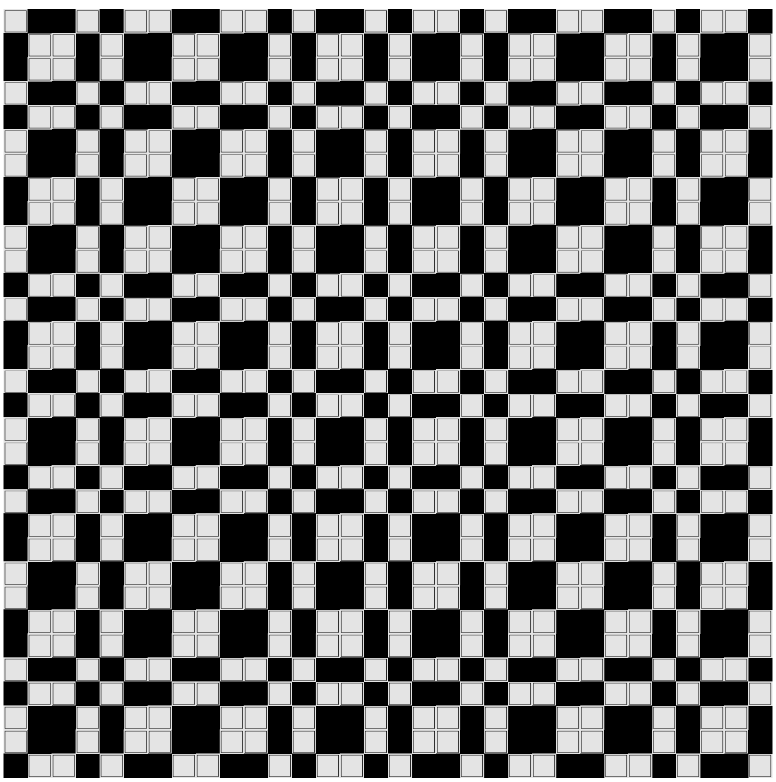
D-Generation N	L-Generation N
L-Generation N	D-Generation N

In the next part, I'll show how to construct variations on the Morse-Thue Patterns when we have squares with more than two colors. To conclude, let's look at the "5" and the "6" generations.

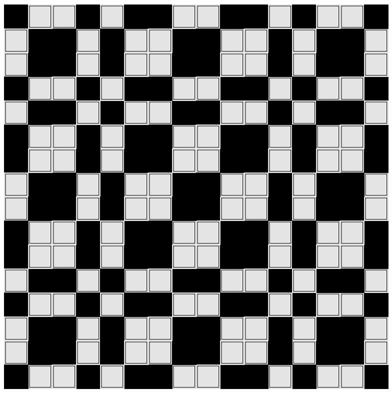
L-Generation 5:



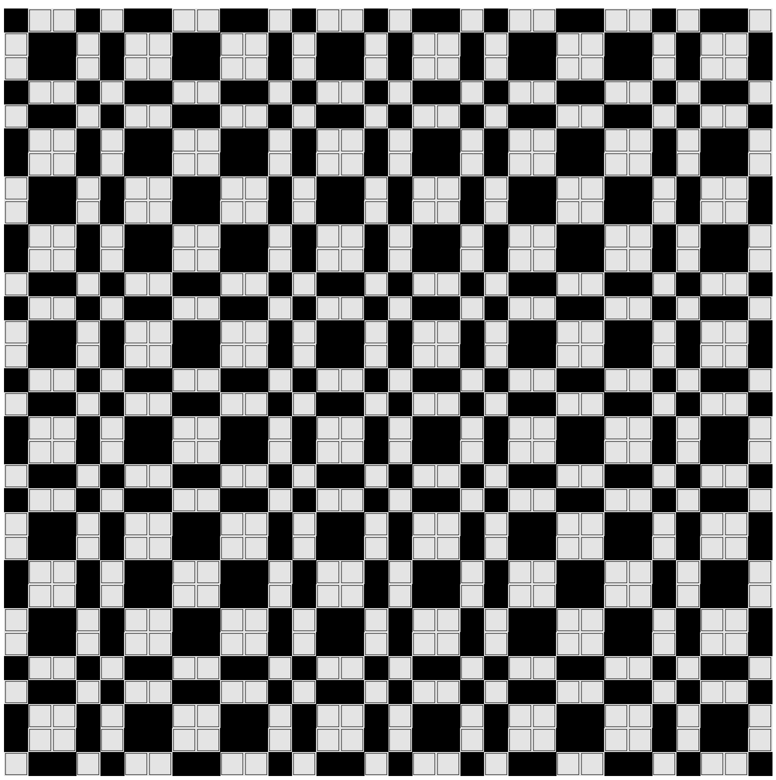
L-Generation 6:



D-Generation 5:



D-Generation 6:



1. The On-Line Encyclopedia of Integer Sequences,
http://www.research.att.com/~njas/sequences/

2. Weisstein, Eric W. "Thue "Thue-Morse Sequence." From *MathWorld*--A Wolfram Web Resource.
http://mathworld.wolfram.com/Thue-MorseSequence.html

3. Griswold, Ralph E. *The Morse-Thue Sequence*, 2001,
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_mt.pdf