

# Block Substitution, Part 2: Initial Patterns, Substitution Blocks, and Color

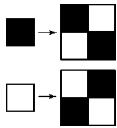
The first article on block substitution [1] mentioned several possible generalizations to the basic scheme. This article explores variations on initial patterns and substitution blocks, as well as color.

## The Initial Pattern

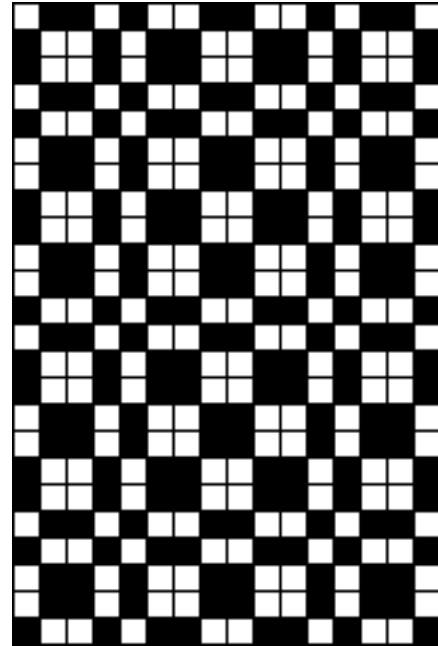
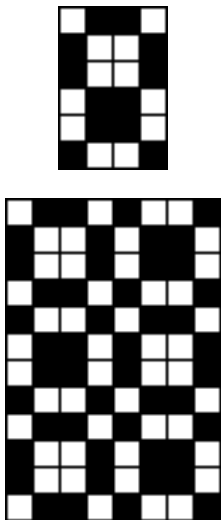
In the basic model of block substitution, the initial pattern is a single cell that is either black or white. It is easy to allow initial patterns that are more complicated, such as



With the block substitution rule

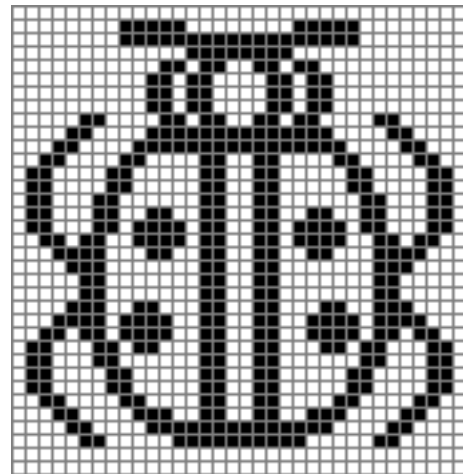


the first few patterns are:

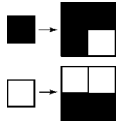


This is a variation on the Morse-Thue carpet [1].

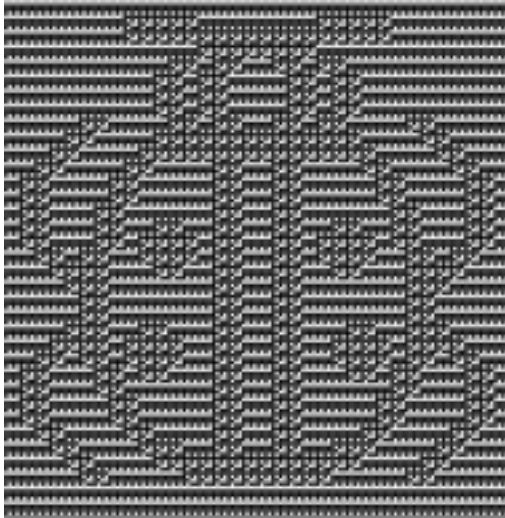
The initial pattern can be large. Consider this  $34 \times 35$  beetle motif:



With the substitution rule



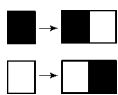
after two iterations, the result (reduced to fit) is



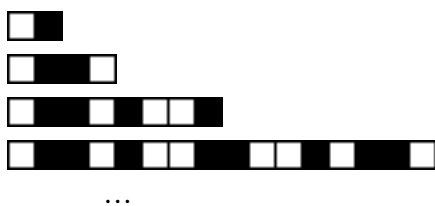
### Substitution Pattern Dimensions

In the first article in this series, the substitution patterns all were  $2 \times 2$ . They can be any rectangular shape as long as the substitution patterns for white and black have the same dimensions.

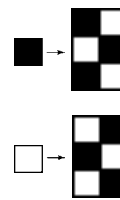
In the extreme case of  $1 \times n$  and  $n \times 1$  substitution patterns, successive patterns develop in only one dimension. For example, the block substitution



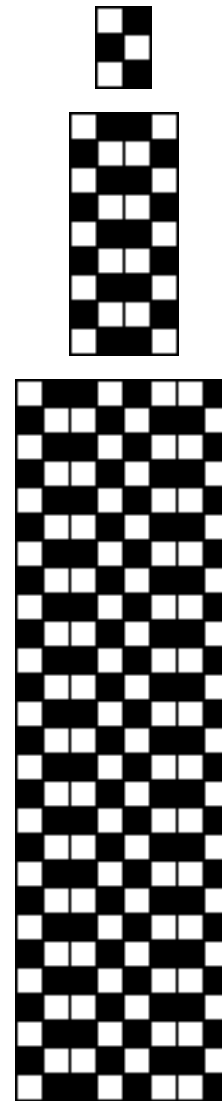
starting with a single white cell, produces the patterns



The dimensions of the substitution patterns determine the rate of increase in size of the resulting patterns. For example, starting with a single white cell, the substitution rule



produces the following results:



The width doubles for each iteration and the height triples.

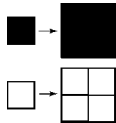
### Substitution Rules with Special Uses

Some substitution rules have special uses. For example, the rule

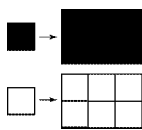


reverses the colors of the initial pattern in one iteration and without changing its size. This is a hard way to accomplish color reversal in practice, but it is worth having in the block-substitution repertoire.

The rule

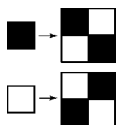


doubles the dimension of the initial pattern without otherwise changing it. There are many other kinds magnifications rules, some of which increase the width and height differently. For example, the rule



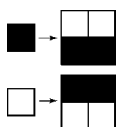
at each iteration triples the width while doubling the height.

Rules such as



break up the initial pattern on successive iterations, producing more detailed patterns on successive iterations.

On the other hand, rules such as

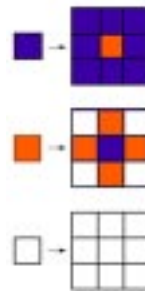


extend the lengths of horizontal lines; their vertical counterparts extend vertical lines.

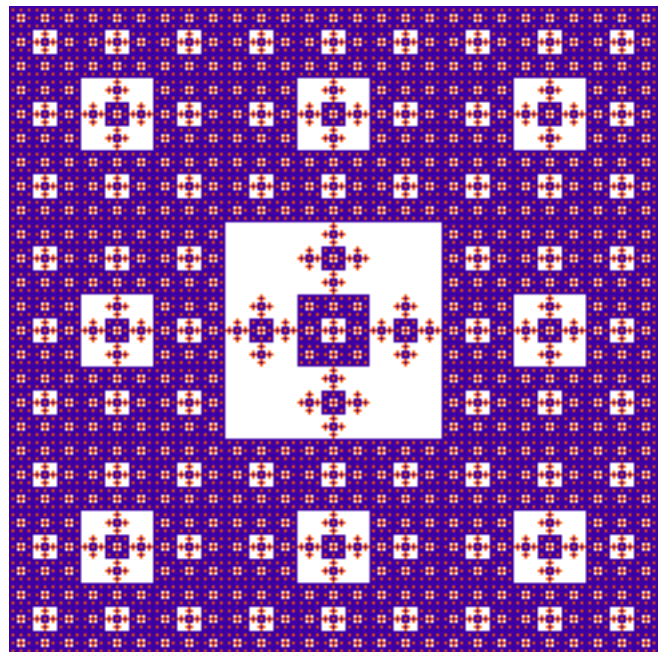
## Color

All of the examples given so far have only two colors. The use of black and white is a matter of convenience; any two different colors could be used.

More colors can be used. Here is an example of a three-color block substitution system:



Starting with a single blue cell, the result after five iterations is.



Multi-color block substitution is a large subject. It will be covered in future articles.

## Reference

1. *Block Substitution, Part 1: Basic Concepts*, Ralph E. Griswold, 2004:

[http://www.cs.arizona.edu/patterns/weaving/webdocs/gre\\_bs1.pdf](http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_bs1.pdf)

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