Creating Weavable Color Patterns, Part 1: Design Methods

A previous article on weavable color patterns [1] described how to determine if a color pattern is weavable and, if so, how to create a draft for it.

This article looks at weavable color patterns from a different perspective: How to create color patterns that are guaranteed to be weavable. This article is a precursor to an article on an interactive application for creating weavable color patterns.

Much of the material that follows is basic and in some cases obvious. We're presenting it here to provide a foundation for what follows.

In this context color patterns are $i \times j$ arrays of colored cells. An array in which every column and row is labeled with a different color is called *nonredundant*.

One question is how many cells are needed to create a weavable pattern that has k different colors. Obviously, this can be done with a $1 \times k$ or $k \times 1$ pattern: a single row or column with a cell for each different color. This case, however, is degenerate and uninteresting.

In general, to have k colors in a nonredundant array, there must be $i + j = k$ columns and rows with different colors assigned to each one. If $i + j > k$, more complex patterns are possible, but the minimum size will do to begin with; the case of redundant arrays with duplicate color assignments later will be addressed later.

For nonredundant arrays, k is partitioned into two parts. There are $k - 2$ non-degenerate size combinations, given by

$$i = k - j \qquad 2 \le j \le k - 2$$

Since these arrays have $i \times j$ cells, the largest array occurs for $i = j$ or $i = j \pm 1$, depending on whether k is even or odd.

Suppose there are eight colors and a 4×4 array as shown in Figure 1.

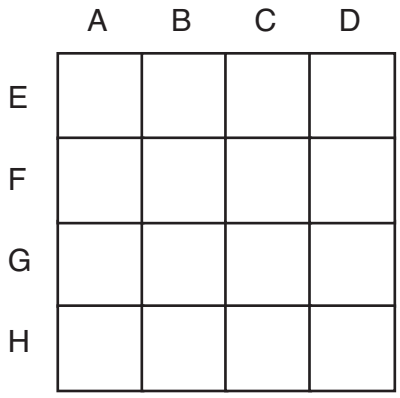

Figure 1. A 4 × 4 Array

It is obvious that it's possible to have all k colors in such an array. Figure 2 shows one such pattern.

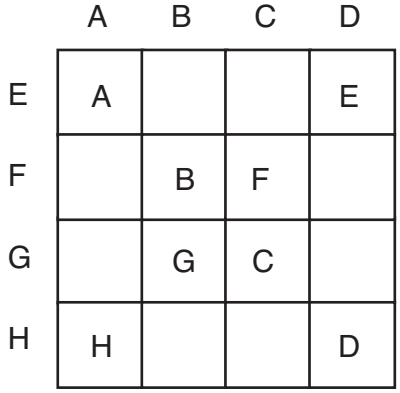

Figure 2. An 8-Color 4×4 Pattern

The remaining cells in Figure 2 can be colored in any of the ways the column and row labels allow. Since there are eight cells of unspecified color, there are $2^8 = 256$ possible patterns based on Figure 2.

For k a multiple of four and $i = j = k / 2$, it is possible to assign colors to cells so that each color occurs $k / 4$ times ($i \times j = k^2 / 4$). Here is a coloring algorithm for constructing such color-balanced patterns:

- For each odd-numbered row, assign alternate cells the column and row colors.
- For each even-numbered row, assign alternate cells the row and column colors.

Figure 3 on the next page shows the result for a 4×4 array.

November 13, 2001; last revised August 1, 2004

	A	B	C	D
E	A	E	C	E
F	F	B	F	D
G	A	G	C	G
H	H	B	H	D

Figure 3. A Balanced 4×4 Color Pattern

For other array shapes, color balance is not possible, but the coloring algorithm given above assures *k*-colored patterns.

The patterns produced by this algorithm can be quite attractive. See Figure 4 for an example.

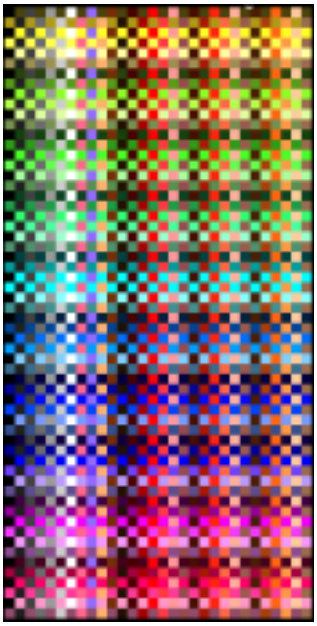

Figure 4. An Algorithmic Pattern

Transformations that Preserve Weavability

Given a weavable color pattern, there are several kinds of changes that can be made to it that preserve weavability:

1. duplicating existing rows and columns
2. deleting rows and columns
3. rearranging rows and columns
4. rotating the pattern in 90° increments
5. flipping the pattern horizontally, vertically, or diagonally
6. adding solid-colored rows and columns

These changes do not require knowledge of the colors assigned to columns and rows. Here are two that do:

7. adding a column whose cells are colored either with the new column color or their corresponding row colors, and similarly for rows
8. setting the color of a cell to the color of its column or row

The first kind of change, duplicating existing rows and columns offers many design possibilities. For example, duplicating adjacent rows and columns can be used to produce bands of any desired width. Mirroring, horiztonal, vertical, or both also follows. Figure 5 shows a weavable color pattern created using only duplications of the rows and columns of an algorithmic pattern:

Figure 5. A Weavable Color Pattern

More to Come

In the next article on weavable color patterns will describe an interactive computer application that allows the user to construct weavable color patterns, but in ways different from those described in this article.

Reference

1. Ralph E. Griswold, "Weavable Color Patterns", 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_cwev.pdf

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