Color Complementation, Part 2: Complementation Sequences

The first article in this series explored color-alternate patterns, in which even-numbered rows, columns, or both are complemented [1].

Generalized Complementation

Complementation is not limited to even numbered lines. It can be done for odd-numbered lines or any specified lines.

The natural way to characterize the lines to be complemented is by a sequence of 0s and 1s, where 0 stands for no complementation (leave the line as it is) and 1 stands for complementation.

For example, complementation even-numbered lines is represented by the sequence 01010101 …, or just 01 with the understanding that it repeats as necessary. Similarly 10 complements odd-numbered lines, and 1100 complements two lines and leaves the next two unchanged, repeating.

In a general model, both rows and columns can be complemented in different ways, so there are two complementation sequences, say r and c. Then the sequence 0, meaning no lines are complemented, can be used if just row or column complementation is wanted. For example, $r = 01$ and $c = 0$ complements even-numbered rows but leaves the columns unchanged. Note also that the sequence 1 complements all lines and corresponds to turning the pattern over.

This model allows a wide range of color-complementation patterns. A few examples are shown at the end of this article.

Repeats

The problem of patterns with an odd number of lines and color alternation was discussed in the first article in this series in terms of repeating the pattern to get an even number of lines.

The situation is more complicated in the case of sequences whose period is not 2.

The number of repeats of the pattern in the general case is

$$lcm(lines, period) \;/\; lines$$

where *lines* is the number of lines in the pattern (rows or columns) and *period* is the length of the period in the corresponding sequence. ($lcm(i, j)$ stands for the least common multiple of i and j; the smallest number that both i and j divide evenly.)

For example, with a pattern of 7 rows and the row sequence 10,

$$lcm(7, 2) = 14$$

so the number of row repeats needed is 12 / 7 = 2.

On the other hand, with a pattern of 8 rows and the row sequence 110100,

$$lcm(8, 6) = 24$$

and the number of row repeats needed is 24 / 8 = 3.

Possibilities

As is usual with a subject like this, the possibilities are unlimited: there is no limit to the number of possible complementation sequences and they can be used in three combinations for every pattern considered.

The kind of patterns that result from color complementation depend strongly on the kind of pattern to which they are applied.

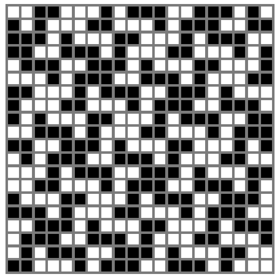
Future articles in this series will explore color complementation for different kinds of patterns, such as crepes, fancy twills, and crackle weaves.

Reference

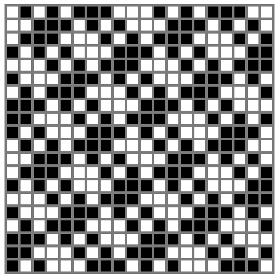
1. *Color-Complementation, Part 1: Color-Alternate Weaves*, Ralph E. Griswold, 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_com1.pdf

Ralph E. Griswold

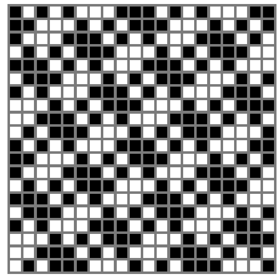
Department of Computer Science
The University of Arizona
Tucson, Arizona



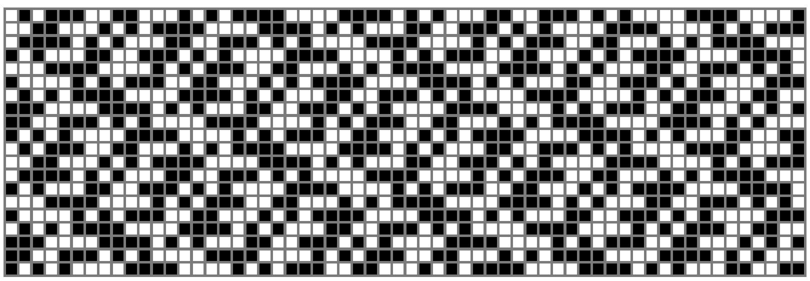
original crepe pattern



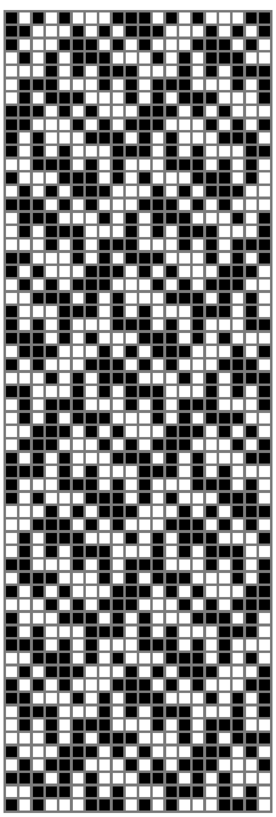
r = 0110 *c*= 0110



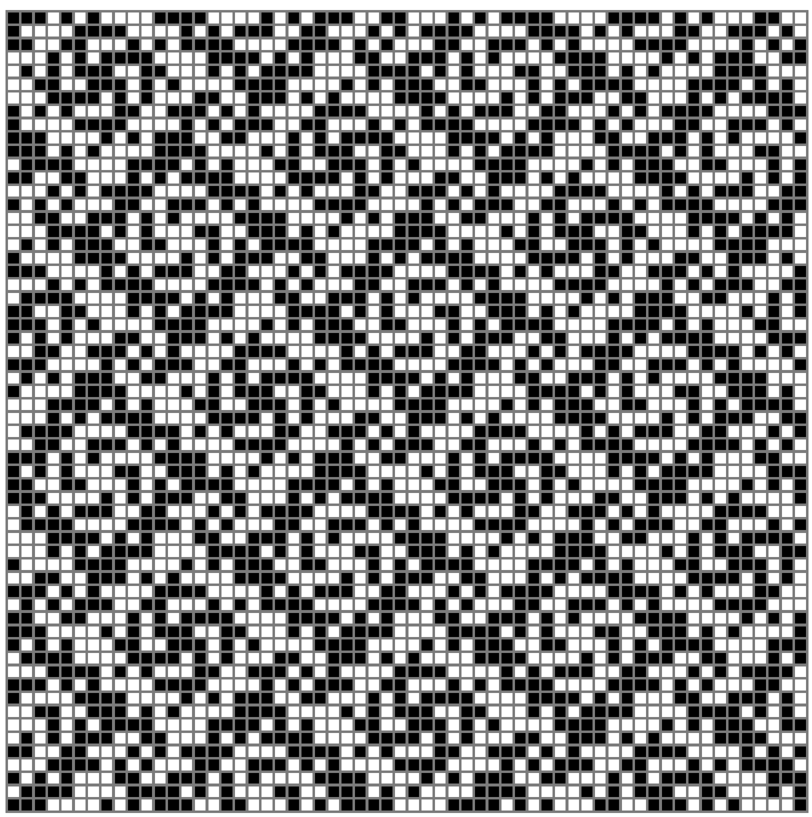
r = 0110 *c*= 1001



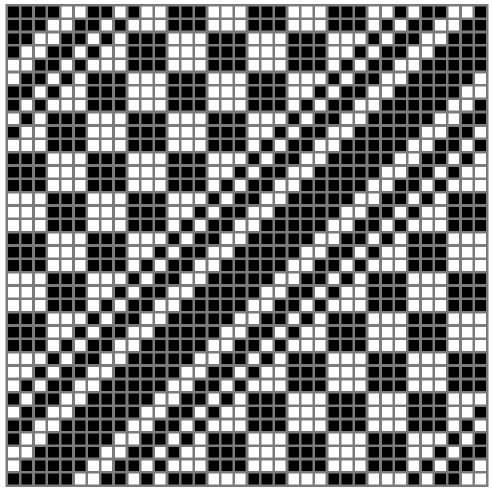
r = 0110 *c*= 011



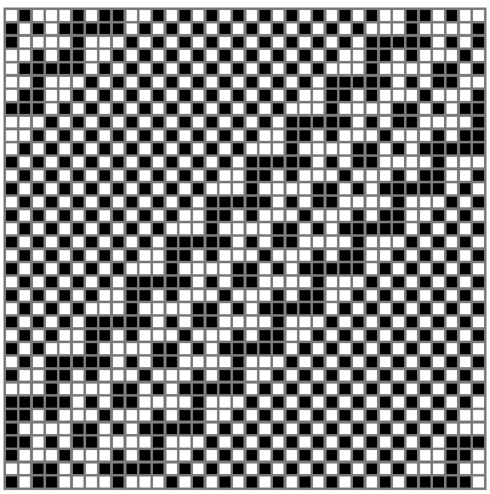
r = 011 *c*= 1001



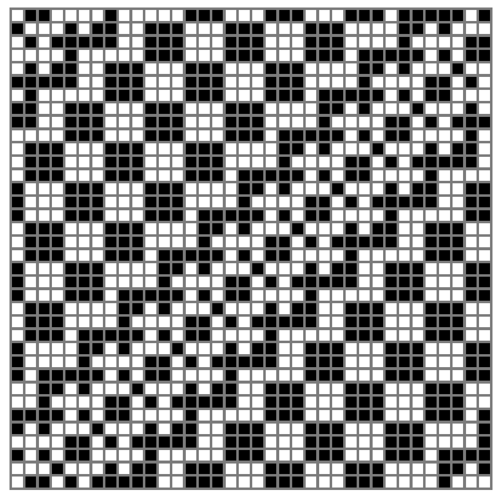
r = 011 *c*= 110



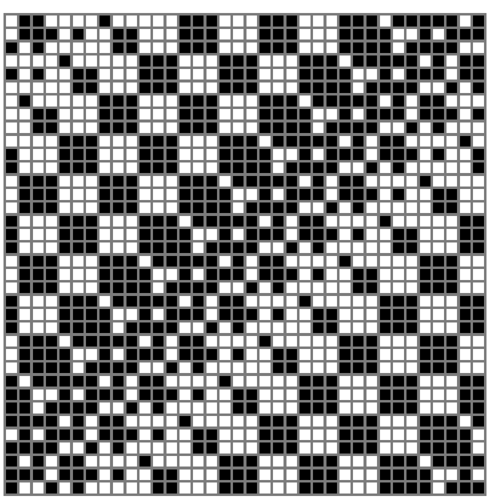
original fancy twill



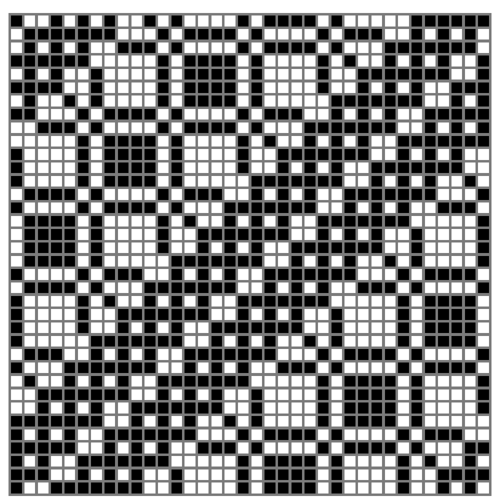
$r = 001 \;\; c = 101$



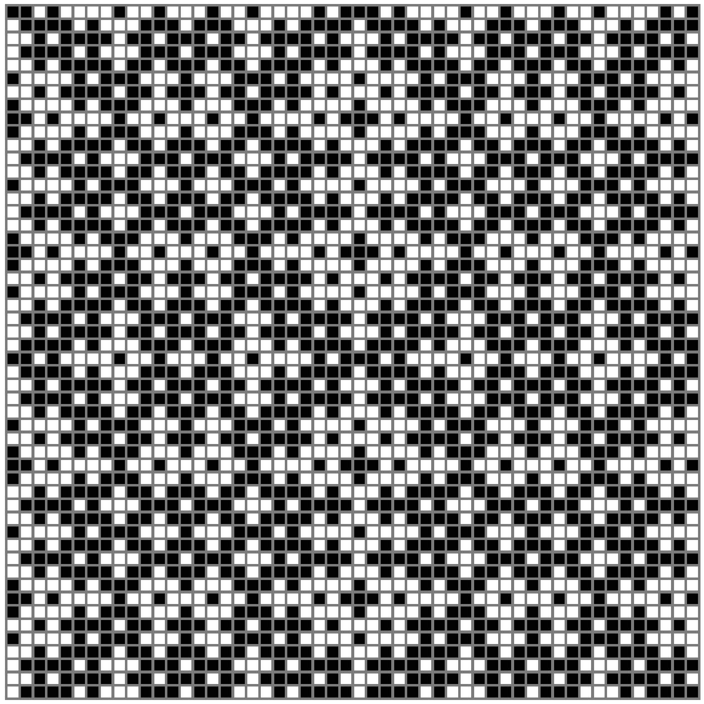
$r = 101 \;\; c = 011$



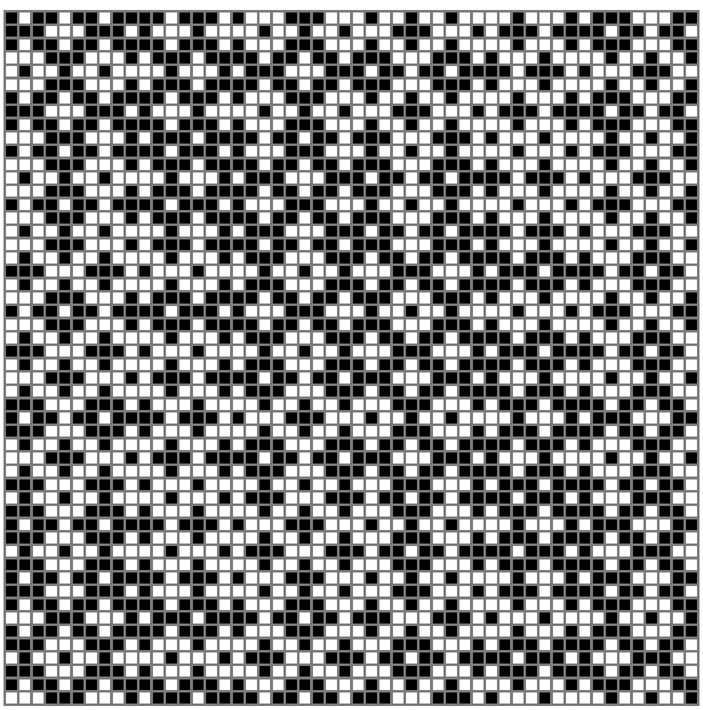
$r = 110 \;\; c = 011$



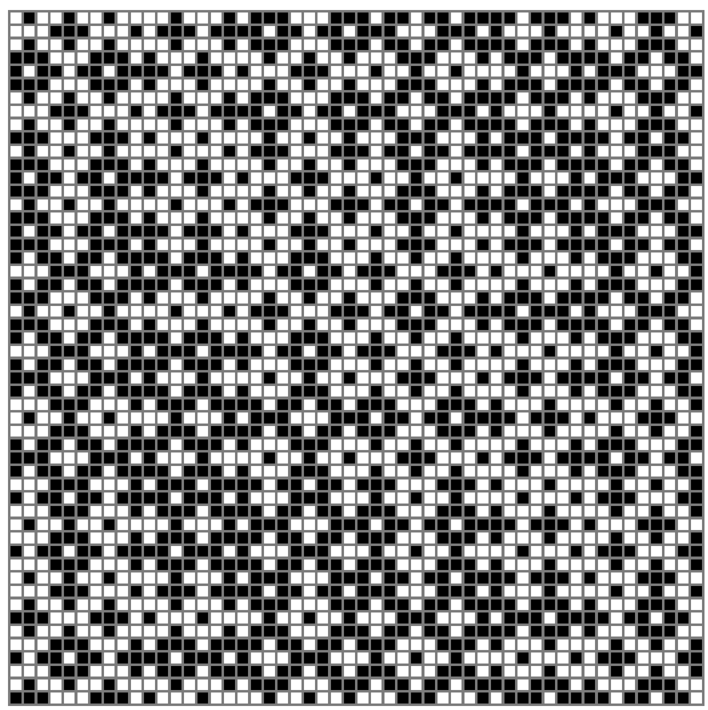
$r = 0110 \;\; c = 0110$



original crackle weave



r= 0110 c = 0110



r= 0110 c = 1001