# Constrained Patterns, Part 1: Basic Concepts

*Note:* This article was inspired by material in Stephen Wolfram's remarkable book, *A New Kind of Science* [1].

Constraints limit what is possible. With respect to interlacement patterns, constraints impose both color and structural limitations. Constraints can take many forms. Expressed in terms of drawdowns, examples are:

1. The number of white cells and black cells must be equal.
2. No more than four consecutive cells in any row and column can be the same color.
3. Every cell must have at least one adjacent cell of the opposite color.
4. Constraints 1, 2, and 3 all must be satisfied.

Constraint 1 is a global constraint and is equivalent to requiring that a weave be balanced. This constraint cannot be satisfied by a drawdown with an odd number of cells. That is, of all drawdowns, only ones with even dimensions can possibly satisfy this constraint.

Constraint 2 is more local and in more familiar terms limits float length.

Constraint 3 is local; it specifies a properly that all neighborhoods must have. (See the articles on drawdown automata [2-3].)

Constraint 4 requires that three constraints be simultaneously satisfied. It is called a *constraint set*. In this sense, Constraints 1, 2, and 3 are constraint sets containing only one constraint.

## Constraint Analysis

Given a pattern, it generally is easy to determine if it satisfies a given constraint set. For example, whether or not a pattern satisfies Constraint Set 1 can be determined just by counting black and white cells. Similarly, whether or not a pattern satisfies Constraint Set 2 can be determined by examination or using the float-analysis feature of a weaving program.

Constraint Set 3 requires a little more work, since it may be necessary, in general, to examine a large number of individual neighborhoods.

And, of course, determining whether or not a pattern satisfies Constraint Set 4 can be determined by checking each of its constraints.

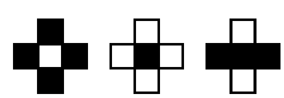
It is important to realize that there are constraint sets that no patterns satisfy. For example, a constraint set that contains Constraint 1 and a constraint that patterns must have an odd number of cells cannot be satisfied — it is *unsatisfiable*. In this example, it is obvious that the two constraints are mutually exclusive. In general, it may be difficult to determine whether or not a constraint set can be satisfied any pattern.

## Neighborhood Constraints

As far as weave structure is concerned, neighborhood constraints are interesting, since they have a strong effect on appearance.

Neighborhood constraints can be characterized by *neighborhood templates*. As with drawdown automata, there are many kinds of neighborhoods that can be used. We'll use the von Neumann 5-cell neighborhood [2] in what follows. This neighborhood is small enough to be computationally tractable but large enough to characterize a wide range of structural characteristics.

Neighborhood constraints can be pictured like this:



None of these constraints *taken alone* is satisfiable, simply because they all require every cell to be the same color (white in the first, black in the other two) while simultaneously requiring that them to be surrounded by cells



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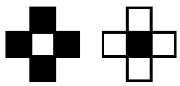
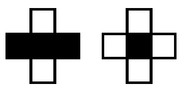
of other colors.

Taken in combination in constraint sets, however, they may be satisfiable. For example, the constraint set consisting of the templates



is satisfied by plain weave (and only plain weave).

On the other hand, the constraint set consisting of the templates



is unsatisfiable because it requires every cell to be black and at the time to have adjacent white cells.

Neighborhood constraints can be looked at in several ways:

- Does a pattern satisfy a given neighborhood constraint set?
- What neighborhood constraint set does a given pattern satisfy.
- What patterns satisfy a specific neighborhood constraint set?

The first question is easy to answer: as mentioned above, it's only necessary to compare the cell neighborhoods to the templates in the constraint set.

The second question also is generally easy to answer by cataloging the neighborhoods of all cells, although there are some issues to be addressed, such as how to handle cells at the borders that do not have complete neighborhoods.

The third question is, in general, much harder to answer. It is, nonetheless, interesting. For example, it would be interesting to know what patterns satisfy the same neighborhood constraint set that a 2/2 twill does. The problem is hard because there is no is known way to construct patterns from constraint sets that does not involve a large amount of computation.

## More to Come

In the next article on constrained patterns, we'll look at the question of finding constraint sets for specific patterns. Following that, we'll look at the harder problem of finding patterns that satisfy given constraint sets.

## References


1. Stephen Wolfram*A New Kind of Science*, Stephen Wolfram, Wolfram Media, 2002.

2. *Drawdown Automata, Part 1: Basic Concepts*, Ralph E. Griswold:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_dda1.pdf

3. *Drawdown Automata, Part 2: Neighborhoods and State-Transition Rules*, Ralph E. Griswold:
http://www.cs.arizona.edu/patterns/ weaving/webdocs/gre_dda2.pdf



Ralph E. Griswold

Department of Computer Science
The University of Arizona
Tucson, Arizona