# Drawdown Automata, Part 3: Pattern Sequences

As we illustrated in an earlier article [1], drawdown automata can produce sequences of interesting patterns. In this article we'll address pattern sequences (but not whether they are interesting in the context of weave design).

The nature of pattern sequences depends on the size of the automaton, the neighborhood used, and the state-transition rule.

The number of possible patterns is determined by the size of the automaton. For an automaton $n$ cells wide and $m$ cells high, there are $2^{n \times m}$ possible patterns. This number becomes huge very quickly as $n$ and $m$ increase. Table 1 shows some values for square automata.

| *n* | *m* | *patterns* |
|---|---|---:|
| 3 | 3 | 512 |
| 4 | 4 | 65,536 |
| 5 | 5 | 33,554,432 |
| 6 | 6 | 68,719,476,736 |
| 7 | 7 | 562,949,953,421,312 |
| 8 | 8 | 18,446,744,073,709,551,616 |

**Table 1. Number of Patterns for Square Automata**

The *successor* of a pattern $P$ is the pattern $P'$ that results from applying the state transition rule to $P$. Conversely, $P$ is the *predecessor* of $P'$. A pattern, of course, has exactly one successor. All patterns have successors but not all patterns have predecessors. Patterns without predecessors are called *outer patterns*. If applying the state transition rule to $P$ does not change $P$ (that is if $P = P'$), then $P$ is called a *terminal pattern*.

Repeatedly applying the state-transition rule to a pattern $P$ results in a *pattern sequence*, $P, P', P'', \ldots$ . Since there are only finitely many patterns, pattern sequences eventually fall into a loop. In some cases, the sequence may lead back to $P$ itself and the entire sequence is a loop. In other cases, the sequence may repeat starting with a later pattern in the sequence. Patterns in a loop are called *cycle patterns*. Note that terminal patterns are cycle patterns. In the parlance of cellular automata, loops are called *basins of attraction*.

Figure 1 shows a typical cycle pattern sequence. The eighth pattern is the same as the first.

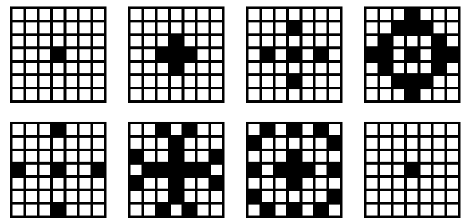

**Figure 1. Cycle Sequence**

Patterns that lead to a loop but are not in a loop are called *lead-in patterns*.

Figure 2 shows the pattern sequence for a different rule. The first pattern has no predecessor and is an outer pattern. The first 17 patterns constitute a lead-in sequence for a terminal pattern.

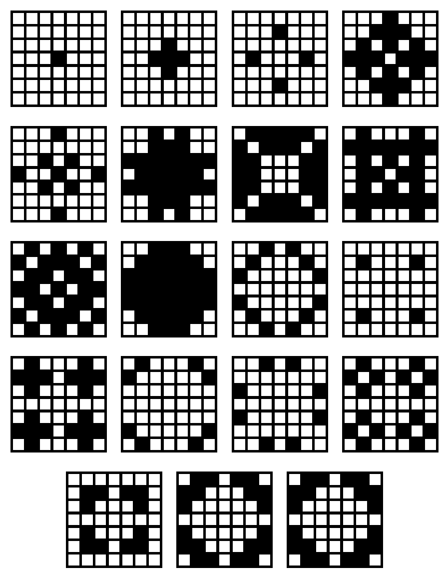

**Figure 2. Sequence Leading to Terminal Pattern**

The situations that can arise in pattern sequences are more easily understood using *state-transition diagrams*. Figure 1 on the next page shows an example.



February 23, 2002; last modified August 1, 2004

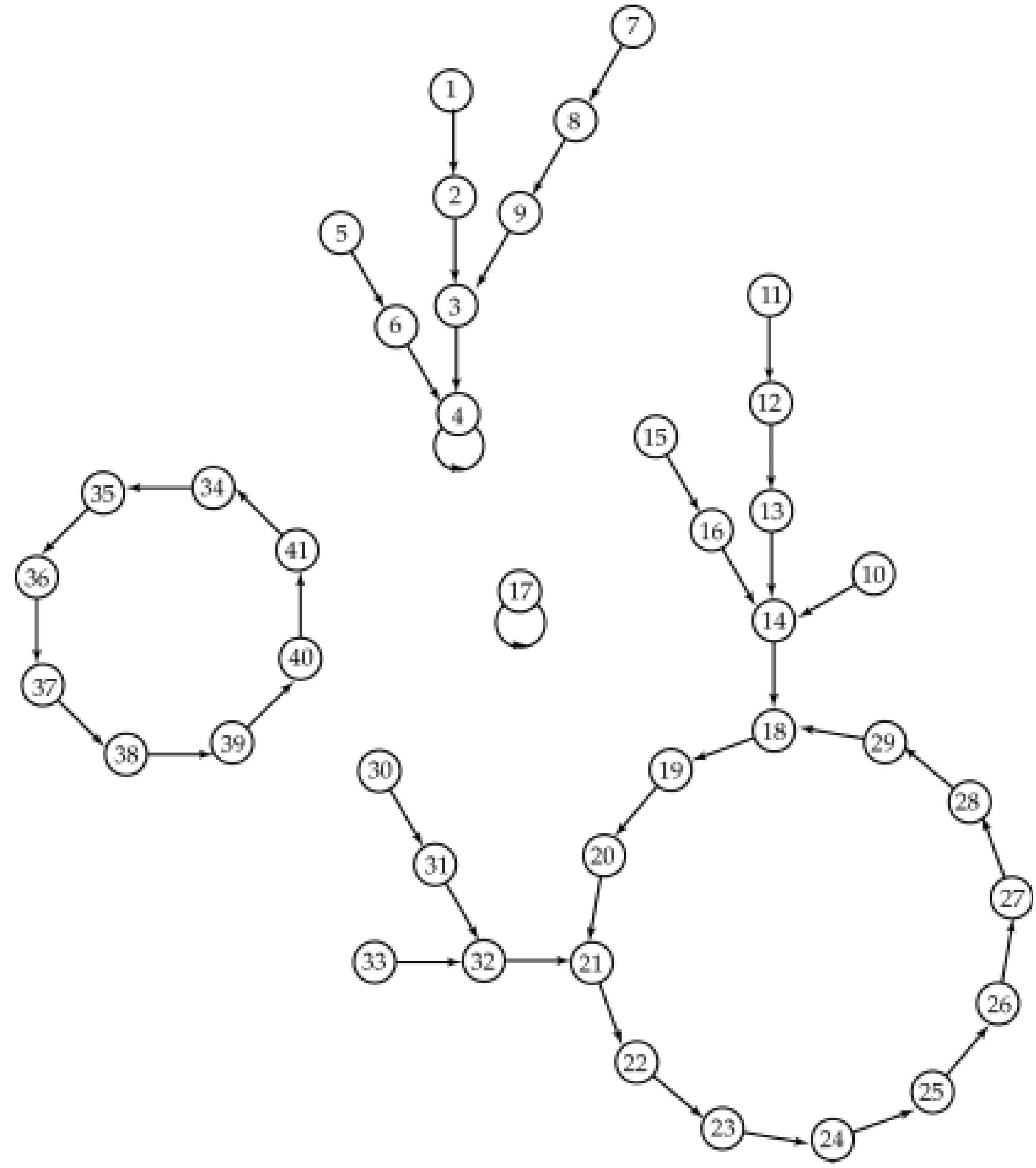


**Figure 1. A State-Transition Diagram**

In this state-transition diagram, patterns $P_1$, $P_5$, $P_7$, $P_{10}$, $P_{11}$, $P_{15}$, $P_{30}$, and $P_{33}$ are outer patterns. $P_4$ and $P_{17}$ are terminal patterns. $P_1$ through $P_3$, $P_5$ through $P_{16}$, and $P_{30}$ through $P_{33}$ are lead-in patterns. $P_{17}$, $P_{18}$ through $P_{29}$, and $P_{34}$ through $P_{41}$ are cycle patterns.

What would a complete state-transition diagram look like? Except for very small patterns, no one knows. Some parts of some automata for some rules have been explored, but the general problem for automata of even modest size is intractable. And how would you draw a diagram for even thousands of patterns? There is a vast uncharted territory.

For the purpose of finding interesting patterns for weave design, rules that produce long pattern sequences have the most potential.

In the next article on drawdown automata, we will explore a few rules that not only have long pattern sequences but also are known to produce visually attractive patterns of the kind useful in weave design.

## Reference

1. *Drawdown Automata, Part 1: Basic Concepts*, Ralph E. Griswold, 2002:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_dda1.pdf

Ralph E. Griswold
Department of Computer Science
The University of Arizona
Tucson, Arizona