Silliness: Gödel-Numbering Drawdowns

The subject of this article is pure silliness. It has no practical use. Be warned.

The idea is to find a way to assign a unique number to every drawdown. Whether or not it might be useful to be able to identify every drawdown numerically, the method used here, as you'll see, is nonsense in the extreme. It's just an exercise in a little funky mathematics.

The idea is based on prime numbers (those numbers that have no divisors but 1 and themselves). The first few prime numbers are 2 (the only even prime), 3, 5, 7, 11, … .

The fundamental theorem of arithmetic states that every positive integer has a unique representation as a product of primes raised to powers:

$$2^a \times 3^b \times 5^c \times 7^d \times \ldots$$

where a, b, c, d, … are nonnegative integers. Since these integers apply to successive primes, they alone are enough to characterize a number. Of course, we can stop when all subsequent exponents are 0.

For example, here are a few integers and their representations as powers of primes:

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$10000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 = 2^4 \times 3^0 \times 5^4$$

$$11011 = 7 \times 11 \times 11 \times 13 = 2^0 \times 3^0 \times 5^0 \times 7^1 \times 11^2 \times 13^1$$

Expressed as just sequences of exponents, these numbers are:

36 = {2, 3}

10000 = {4, 0, 4}

11011 = {0, 0, 0, 1, 2, 1}

Conversely, if we have a sequence of exponents, we can compute the corresponding number, as in

$$\{1, 2, 0, 1, 4\} = 2^1 \times 3^2 \times 5^0 \times 7^1 \times 11^4 = 1844766$$

To get the exponents back from the number, all that's necessary is to factor the number. This can be done by dividing by successive primes. The number of times a prime divides the number evenly is the desired exponent.

What does this have to do with drawdowns?

A drawdown can be represented as an array of ones and zeros — ones for black cells, zeros for white cells (say). Thus the drawdown in Figure 1 can be represented by the array in Figure 2.

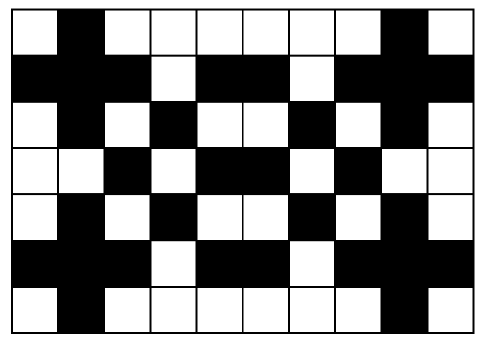

Figure 1. A Drawdown

0100000010
1110110111
0101001010
0010110100
0101001010
1110110111
0100000010

Figure 2. A Drawdown Array

Okay, now what? If we string the rows of the array together, one after the other, we get a result that looks like this:

0100000010111011011101010010100010110100010100101011101101110100000010

We can interpret this number as a sequence of exponents for the product of primes (each exponent being either a 0 or a 1):

{0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, …}

Multiplying out the primes raised to these powers, the result is:

128083849210783394269323498771322 70

April 24, 2002; last modified August 11, 2004

4266130671612759323864378761305O3

A big number, indeed.

But there’s a problem: We need to include, somehow, the dimensions of the drawdown — its width (number of columns) and its height (number of rows). For Figure 1, the width is 10 and the height is 7.

We’ll put these numbers at the beginning of the exponent sequence as the powers of 2 and 3, shifting the exponents for the array two places, so that they start with the power of 5 instead of the power of 2:

{10, 7, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 0, …}

Multiplying this out, we get

2868422433013588820654187436163919403315484455087151526823898633974790
2464

This number is, of course, $2^{10} \times 3^{8} = 2239488$ times larger than the result without the dimensions included. A big price to pay for a couple of small numbers.

Well, we told you this article was silly.

Why do we call this Gödel numbering? It’s because the mathematician Kurt Gödel used this kind of technique to prove one of the most remarkable and unexpected mathematical results of all time. See the side-bar.

If you’ve thought about what we did, you’ve probably noted there are much simpler ways to assign unique numbers to patterns. For example, when we strung the rows of the array together, we got a result that could be interpreted as a number in the base 2. Converted to base 10, the result is:

2985248420324315537 94

This is a much smaller number than the Gödel number, although the method isn’t nearly as esoteric. This method doesn’t include the dimensions, however. Can you figure out a way to do this?

Gödel and Undecidability

Kurt Gödel 1906 - 1978

Kurt Gödel shook the foundations of mathematics by proving that certain mathematical propositions cannot be proven to be true or false.

Put in an informal way, Gödel’s incompleteness theorem states that all consistent axiomatic formulations of arithmetic include undecidable propositions [1].

The method Gödel used relied on a system by which every mathematical proposition was represented by a distinct number. Such numbers are called Gödel numbers. For example, the proposition (∃x)(x = sy), which means “there exists an x such that x is the immediate successor of y” is encoded as

$$2^{8} \times 3^{4} \times 5^{13} \times 7^{9} \times 11^{8} \times 13^{13} \times 17^{5} \times 19^{7} \times 23^{16} \times 29^{9}$$

where the exponents correspond to the symbols in the proposition [1].

Needless to say, this is a very large number:

74880654697373651627226805069425599081
28930612274430799531084603348039652271
735661562500000000

For more information about Gödel’s work and its ramifications, see Reference 2.

References

1. *CRC Concise Encyclopedia of Mathematics*, Eric W. Weisstein, Chapman & Hall/CRC, 1999, pp. 741-742.

2. *Gödel, Escher, Bach: an Eternal Golden Braid*, Douglas R. Hofstadter, Basic Books, 1979.