When a Fabric Hangs Together (Or Doesn't)

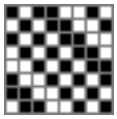
There are two parts to conventional fabric analysis [1]:

1. Determining the interlacement of the warp and weft threads.

2. Producing a draft from this interlacement.

The two parts may be done separately or in combination, depending on the particular technique used. For the purposes of this article, it is convenient to view them separately with the first part producing a "drawdown" pattern.

Drawdowns can be represented in various ways. For visual understanding, a rectangular grid of cells, with each cell representing a point of interlacement, works best. In a warp-faced drawdown, black grid cells indicate where a warp thread is on top and white cells indicate where the weft threads are on top. Here is an example:



drawdown

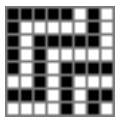
A drawdown that accurately represents the interlacement of a sound fabric can be used to "draw up" a draft for weaving the fabric.

This draw-up process can be used for producing drafts from patterns obtained from sources other than fabric analysis. Some weaving programs provide this capability. But there may be a problem.

A Problem

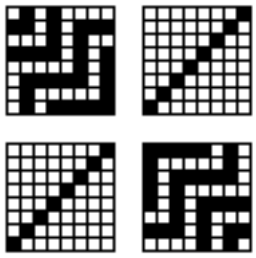
In their seminal paper on weave structures [2], Grünbaum and Shephard of tiling and pattern fame [3], pointed out that patterns that look perfectly reasonable may not produce interlacements that "hang together"; if woven, some interlaced warp and weft threads may not be interlaced with the rest of the fabric. A fabric woven based on such a pattern would come apart in pieces. Such an interlacement is *unsound*.

Consider this pattern and the corresponding drawn-up draft:



pattern

tie-up *threading*



treadling *drawdown*

draft

At first glance, the draft looks perfectly reasonable. If woven, however, the result would be pieces that would not hang together.

What distinguishes patterns whose corresponding interlacements are not sound from ones that are?

Interlacements with long floats, may, of course, not produce stable fabrics, but they hang together, however loosely. Patterns with complete rows or columns of cells of the same colors obviously have unsound interlacements. But there seems nothing obvious about the pattern and draft shown above that would indicate a problem.

Determining whether or not a pattern when drawn up represents an interlacement that hangs together — or doesn't — cannot be done by simple visual inspection. Instead, an algorithm (procedure) is needed. Grünbaum and Shephard's paper was followed by several papers giving algorithms for determining whether or not a fabric hangs together [4-6].

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These algorithms are written in the language of mathematics and are not easy to follow, even for an educated layperson.

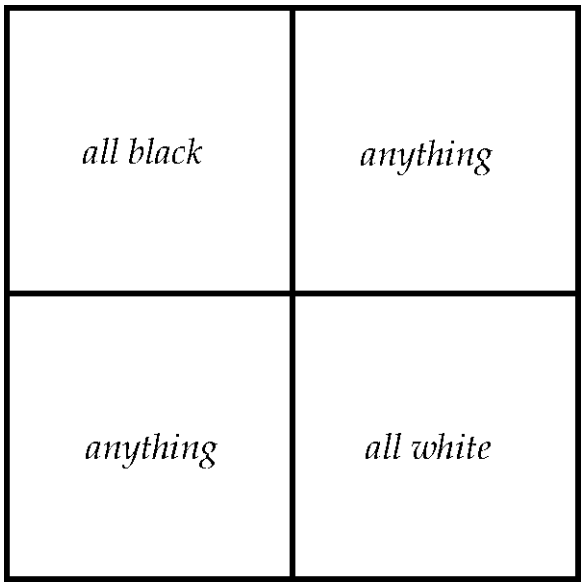
A Mathematical Method

The following procedure is due to Clapham [4].

First note that the columns and rows of a drawdown can be rearranged without affecting whether or not the fabric hangs together.

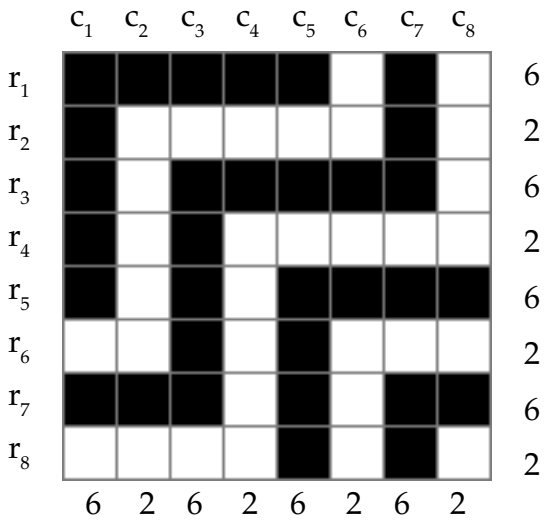
The basic idea is to rearrange the rows and columns so that the bulk of the black (warp) cells are at the top right and the bulk of the white (weft) cell are at the bottom left.

If the resulting pattern can be divided up in this fashion

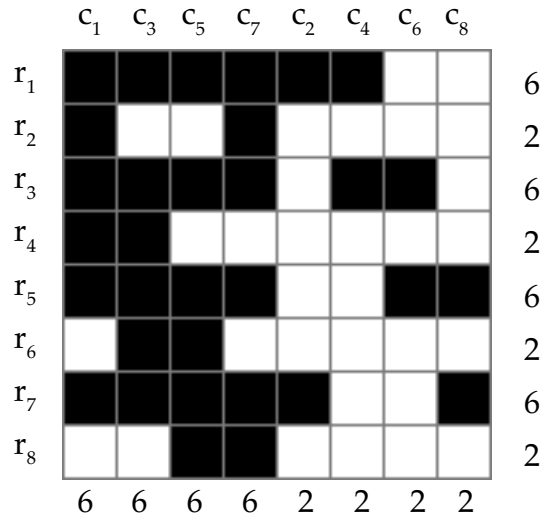


then the fabric would not hang together. The black and white areas need not be square; they can be any rectangular shape, all that is required is that their corners touch as shown.

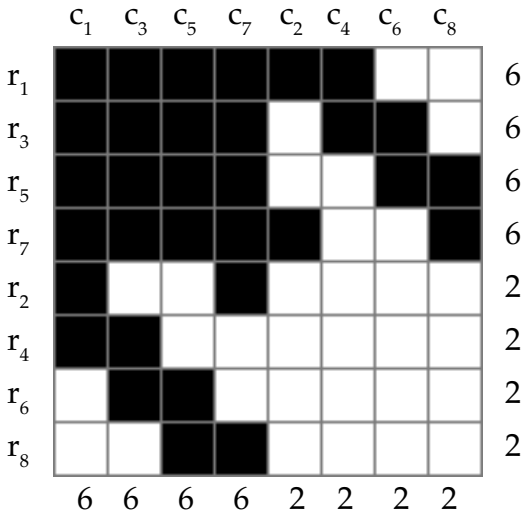
The method of rearranging the rows and columns is first to rank them according to the number of black cells they have. For the example above, this is



The next step is to rearrange the columns so that they are in order of their ranks. If more than one column has the same rank, as in this example, it does not matter in what order the columns with the same rank are placed.



The last step is to rearrange the rows so that they are in the order of their ranks:



The red lines show that the all black and all white blocks meet as required, so the fabric does not hang together. Had they not so met, the fabric would hang together.

An Alternative Method

An alternative method, described by Grünbaum and Shephard [2], is to lift off a thread, lift off what it lifts, and so on. If all threads are lifted off by this process, the fabric hangs together. If not, it doesn't.

This can be done methodically as follows.

First, if there is a solid-colored row or column, it represents a thread that is not interlaced at all and obviously the fabric doesn't hang together.

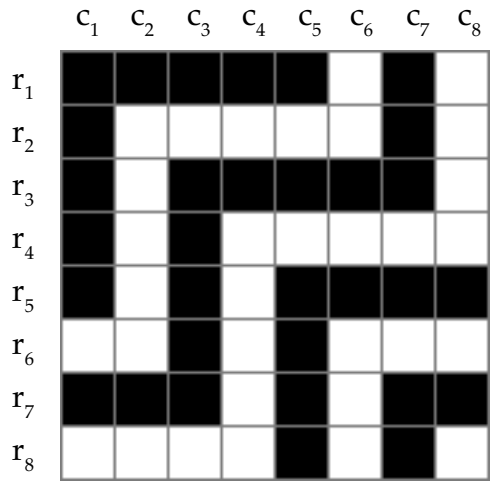
Otherwise, two lists are needed: one to keep track of threads that have been lifted and another to keep track of threads that have been lifted but not checked to see what they lift.

1. Start by putting the first column (warp thread) on the two lists. (Any other thread could be used to start.)
2. If the list to be checked is empty, go to Step 4. Otherwise pick a thread, remove it, and continue to Step 3 with this thread.
3. (a) If the thread is a warp thread, put all the weft threads that are on top of it (white cells in the column) but not already lifted on both lists.

   (b) If the thread is a weft thread, put all the warp threads that are on top of it (black cells in the row) but not already lifted on both lists.

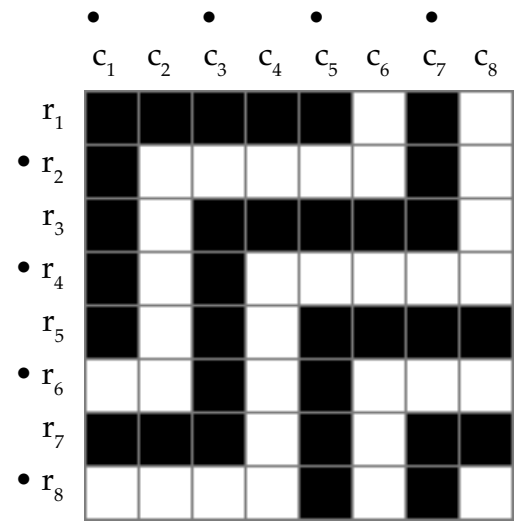
   Go to Step 2.
4. If all warp and weft threads have been lifted, the fabric hangs together. If not, the lifted threads come off the rest and the fabric does not hang together.

Here is how this procedure works on the previous example.



step	*thread*	*to check*	*lifted*
1	c_1	c_1	c_1
2	c_1		c_1
3	c_1	$r_6\, r_8$	$c_1\, r_6\, r_8$
2	r_6	r_8	$c_1\, r_6\, r_8$
3	r_6	$r_8\, c_3\, c_5$	$c_1\, c_3\, c_5\, r_6\, r_8$
2	r_8	$c_3\, c_5$	$c_1\, c_3\, c_5\, r_6\, r_8$
3	r_8	$c_3\, c_5\, c_7$	$c_1\, c_3\, c_5\, c_7\, r_6\, r_8$
2	c_3	$c_5\, c_7$	$c_1\, c_3\, c_5\, c_7\, r_6\, r_8$
3	c_3	$c_5\, c_7\, r_2$	$c_1\, c_3\, c_5\, c_7\, r_2\, r_6\, r_8$
2	c_5	$c_7\, r_2$	$c_1\, c_3\, c_5\, c_7\, r_2\, r_6\, r_8$
3	c_5	$c_7\, r_4$	$c_1\, c_3\, c_5\, c_7\, r_2\, r_4\, r_6\, r_8$
2	c_7	r_4	$c_1\, c_3\, c_5\, c_7\, r_2\, r_4\, r_6\, r_8$
3	c_7	r_4	$c_1\, c_3\, c_5\, c_7\, r_2\, r_4\, r_6\, r_8$
2	r_4		$c_1\, c_3\, c_5\, c_7\, r_2\, r_4\, r_6\, r_8$
3	r_4		$c_1\, c_3\, c_5\, c_7\, r_2\, r_4\, r_6\, r_8$
4			$c_1\, c_3\, c_5\, c_7\, r_2\, r_4\, r_6\, r_8$

Since not all threads have been lifted, the fabric does not hang together. The pattern with the lifted threads marked is



Another Result

A theorem due to Clapham [7] is of interest:

If every weft thread passes both over and under more than one quarter of the warp threads, and if every warp thread passes both over and under more than one quarter of the weft threads, then the fabric hangs together.

The converse is not true; there are many fabrics, notably satins, the hang together but do not meet the requirements of this theorem.

Here are the required over/under numbers to assure a fabric hangs together for some warp/weft threads counts:

threads	*required*
4	2
5	3
6	3
7	3
8	3
9	4
10	4
11	4
12	4
13	5
14	5
15	5
16	5

Creating Interlacements that Don't Hang Together

Note that it is easy to create unsound interlacements. Using the ideas in the mathematical method, just create a pattern with all-white and all-black blocks that meet as required, fill in the other two blocks in any fashion, as long as they are not all black or all white, and rearrange the rows and columns to hide the problem. Why you would want to do this, except to create examples for articles like this, is another question.

More to Come

The interlacement obtained by drawing up from a pattern assumes black warp threads and white weft threads. In cases where this interlacements not sound, it may be possible to find another interlacement that is sound by using color-and-weave effects [8], in which both black and white threads can be used in the warp and weft,.

This will be the subject of a future article.

References

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7. "The Bipartite Tournament Associated with a Fabric", C. R. J. Clapham, Discrete Mathematics, 1985 (57), 195-197

8. *Color-and Weave II*, Margaret B. Windeknecht, self published, 1994.

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