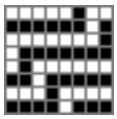
Line-Based Patterns, Part 1: Basic Concepts

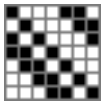
In the context of this series of articles, the term line is used to cover both rows and columns of grid-plot patterns. Grid plots are shown in row order, but columns and rows are equivalent for the subject here.

As such, a line can be represented by a binary sequence of 0s and 1s corresponding to white and black grid cells, respectively. For example, the fifth row in the following pattern has the line sequence 01000000.



Many interesting and important weaves can be made from patterns in which one line serves as a *basis line* and each subsequent line is derived from the former by the application of a *transformation rule*. Such patterns are called *line-based patterns*.

For example, in twills, each line after the basis line is produced by a circular permutation by one of the preceding line:



Transformation Rules

Transformation rules can be of many kinds. The simplest ones are

cyclic permutation

reversal

complementation (exchange of 0s and 1s)

and combinations of these.

For example, given the line

00111011

its cyclic permutation by 3 is

01100111

(Positive values are to the right. Positive values are sufficient, since there is always a positive value that produces the same results as a negative one. For example, –3 for the line above is equivalent to 5.)

The reversal of the given line is

11011100

and its complement is

00100011

Binary Indices

The binary index of a line is the decimal equivalent of the line considered as a binary number.

For example, the binary index for 00111011 is 59.

The combination of a line's length and binary index uniquely characterizes the line. The line length is needed because leading zeros are lost in computing binary indices. For example, 00111011, 111011, and 000000111011 have the same binary index.

The line at the beginning of this section is uniquely identified by 8:59.

Design-Equivalent Patterns

Various transformations of a pattern are *design equivalent* in the sense that they can be derived from each other by simple transformations [1]: rotation in 90° increments, reflections (horizontal, vertical, and around the diagonals), cyclic permutation of rows or columns, complementation, and any combination of these.

Patterns with repeats also are design equivalent to their unit patterns.

Design-Equivalent Basis Lines

For line-based patterns, several different basis lines can produce design-equivalent patterns, which are essentially redundant and need to be avoided. A *fundamental basis line* needs to be selected from the alternatives.

The transformation rules listed above are the ones that produce design-equivalent lines.

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Given a line, basis line can be obtained by applying the transformation in all possible ways and selecting the one with the smallest binary index.

Here is an example of how this can be done. Consider this line

11101100

Its cyclic permutations are

01110110
00111011
10011101
11001110
01100111
10110011
11011001

The one with the smallest binary index is the second, 00111011. (This can be determined by inspection without having to produce all the permutations. For very long lines, the permutations can be sorted as numbers to find the one with the smallest binary index.)

The chosen line now is the basis for applying other transformation. It is not necessary to do this for all the permutations; the results would be the same.

The next transformation is reversal, which produces

11011100

Taking the permutations and selecting the one with the smallest binary index produces

00110111

Since its binary index is smaller than the one for the permutations of the original lilne, it's the one to which the next transformation, complementation, is applied:

11001000

The cyclic permutation of this line that gives the smallest binary index is

00011001

The final transformation is to reverse it to give

10011000

Its cyclic permutation with the smallest binary index is

00010011

so this is the one to use for the basis line. This is the fundamental basis line and has the unique identification 8:19.

For what it's worth, the binary index of the original line is 236.

The Number of Fundamental Basis Lines

Here is a list of the number of fundamental basis lines up to $n = 16$:

n	*lines*
2	1
3	1
4	2
5	3
6	5
7	8
8	14
9	21
10	39
11	62
12	112
13	189
14	352
15	607
16	1144

Soundness

There is no guarantee that line-based patterns will interlacements that "hang together" if drafted by the conventional draw-up technique [2].

This issue must be addressed separately.

Types of Line-Based Patterns

Different transformation rules produce different kinds of patterns (although some patterns produced by one rule may be design equivalent to ones produced by another rule).

The simplest type of line-based patterns use cyclic permutations for successive lines. That will be the subject of the next article in this series.

References

1. *Operations on Patterns, Part 2: Geometrical Transformations*, Ralph E. Griswold, 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_pop2.pdf

2. *When a Fabric Hangs Together (Or Doesn't)*, Ralph E. Griswold, 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_hng1.pdf

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