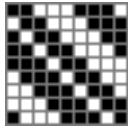


Line-Based Patterns, Part 2: Shift Patterns

The simplest type of line-based pattern [1] involves successive cyclic permutations (shifts) by a fixed amount. Twills, with a shift of 1, are the most familiar of these patterns:



For a basis line of length n , all shifts from 1 to $n-1$ produce patterns (shifts of 0 and n simply repeat the basis line), but it is not necessary to consider all these shifts.

A shift of 1 can be omitted, since the result is a twill and well known. Shifts that are greater than $(n-1)/2$ produce patterns that are design-equivalent [1] to those for shifts that are less than or equal to $(n-1)/2$.

Finally to get square $n \times n$ patterns, a shift must be relatively prime to n ; that is no number other than 1 (and n) must evenly divide n and the shift. Otherwise the vertical repeat will be less than n . While such patterns have potential interest, the focus here is on $n \times n$ patterns. Note that for n prime, all shifts are relatively prime to n .

Here is a list of shifts that meet the requirements above up to $n = 16$:

n	shifts	number
2		0
3		0
4		0
5	2	1
6		0
7	2, 3	2
8	3	2
9	2, 4	2
10	3	1
11	2, 3, 4, 5	4
12	5	1
13	2, 3, 4, 5, 6	5
14	3, 5	3
15	2, 4, 7	3
16	3, 5, 7	3

Note that there are no shift patterns for $n = 6$ (as is the case for satins, and for the same reason).

The first article in this series [1] listed the number of fundamental basic lines up to $n = 16$. Combining that with the information on shifts gives the number of shift patterns:

n	patterns
2	0
3	0
4	0
5	3
6	0
7	16
8	28
9	42
10	39
11	248
12	112
13	945
14	704
15	1821
16	3432

Some examples of shift patterns are given on the last page of this article.

As a final note, all the shift patterns that meet the criteria above produce structurally sound interlacements if drawn up in the conventional manner — they “hang together” [2].

What’s Next?

The next simplest type of line-based pattern involves shifts and complementation for successive lines. That will be the subject of the next article in this series.

References

1. *Line-Based Patterns, Part 1: Basic Concepts*, Ralph E. Griswold, 2004:

http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_lp01.pdf

2. *When a Fabric Hangs Together (Or Doesn't)*, Ralph E. Griswold, 2004:

http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_hng1.pdf

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Line-Based Shift Patterns

