Number Drafting, Part 1: Introduction

Name drafting is a familiar method of creating original weaves [1]. Originally called commemorative drafting, name drafting provides a mechanism for embedding a name or phrase in a weaving, giving that weave a special meaning.

Numbers also can be used in design, again producing original weaves. For the mathematically inclined, or just those fascinated by numbers, searching for numbers that produce interesting designs can be an interesting recreational activity.

Numbers for designs can be selected arbitrarily, created in specific ways, or found among the debris left behind by mathematics.

The process of number drafting, as suggested here, is different from the use of code tables used in name drafting. Here, the digits of a number are taken to correspond to shafts or treadles.

Here is an (arbitrarily chosen) example:

$$3^{77} = 5474401089420219382077155933569751763$$

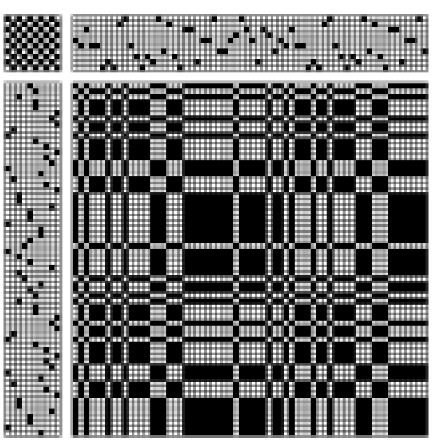
The digits of this number produce the sequence

5, 4, 7, 4, 4, 0, 1, 0, 8, 9, 4, 2, 0, 2, 1, 9, 3, 8, 2, 0, 7, 7, 1, 5, 5, 9, 3, 3, 5, 6, 9, 7, 5, 1, 7, 6, 3

The 0-based representation of a number can be converted to the 1-based system used for identifying shafts and treadles simply by adding 1 to each value in the sequence. This is called *1-adjustment*. The sequence above, 1-adjusted, is

6, 5, 8, 5, 5, 1, 2, 1, 9, 10, 5, 3, 1, 3, 2, 10, 4, 9, 3, 1, 8, 8, 2, 6, 6, 10, 4, 4, 6, 7, 10, 8, 6, 2, 8, 7, 4

With a tabby tie-up, treadled as drawn in, this sequence produces this "3^{77}" design:



Note: Sequences derived from numbers often contain adjacent duplicate values that would create structural problems in weaving. There are various ways of dealing with this problem, such as deleting adjacent duplicates or inserting other values between them. We will not deal with that here.

Base Conversion

Sequences derived from numbers written in the conventional base-10 representation have, of course, at most 10 different values and hence are suitable for 10 shafts and 10 treadles. They can be converted for fewer shafts or treadles by modular reduction [2], but a more natural mathematical approach is to write the numbers in the base that corresponds to the number of shafts or treadles to be used.

For example, 3^{77} written in base 8 is

407452317767256241775612442006762652153 23

and produces the 1-adjusted sequence

5, 1, 8, 5, 6, 3, 4, 2, 8, 8, 7, 8, 3, 6, 7, 3, 5, 2, 8, 8, 6, 7, 2, 3, 5, 5, 3, 1, 1, 7, 8, 7, 3, 7, 6, 3, 2, 6, 4, 3, 4

The base used can be greater than 10. 3^{77} written in base 16 is

41e54cffbab943fdc54880df2d51ad3

where the letters a, b, c, … f stand for the (decimal) values 10, 11, 12, …15. These values can be used in place of the letters in the sequence, which 1-adjusted is

5, 2, 15, 6, 5, 13, 16, 16, 12, 11, 12, 10, 5, 4, 16, 14, 13, 6, 5, 9, 9, 1, 14, 16, 3, 14, 6, 2, 11, 4

For larger bases, additional letters can be used as digits.

Mathematical Note: Base conversion may, of course, change the number of digits in a number and hence the length of the corresponding sequence. Let $\lambda_n(i)$ stand for the number of digits in i when written in base n. Then

$$\lambda_n(i) \approx \lambda_{10}(i) \times ln(10)/ln(n)$$

For example, in converting from base 10 to base 16,

$$\lambda_{24}(i) \approx \lambda_{10}(i) \times ln(10)/ln(16) \approx 0.83 \times \lambda_{10}(i)$$

$\lambda_{10}(3^{77}) = 37$, $\lambda_{16}(3^{77}) \approx 30.73$. The actual value is 31.

Base conversion is relatively simple and can be done by hand. However, other resources are readily available. Some hand-held calculators can convert between bases 10, 8, and 2, although the size of numbers that can be handled is limited. Interactive applets for base conversion can be found on the Web. A few, with various capabilities, are given in Reference 3.

There also are freeware programs for base conversion. Tip: When searching for such resources on the Web, look for both `base conversion` and the synonymous `radix conversion`.

What's Coming Up

Any arbitrarily chosen number of suitable magnitude can be used for number drafting. But the fun lies in finding special numbers that produce attractive designs.

In subsequent articles, we will explore some possibilities. In addition to large numbers in and of themselves, we'll consider subjects such as the decimal (and other base) expansions of fractions (rational numbers) and irrational numbers, which cannot be represented by fractions.

References

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3. Interactive Base-Conversion Resources:
http://www.onlineconversion.com/base.htm
http://fclass.vaniercollege.qc.ca/web/mathematics/real/Calculators/BaseConv_Calc_1.htm
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