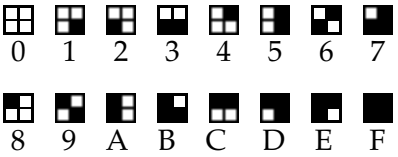
Pantactic Designs: Part 1, 5×5 Pantactic Squares

Basic Concepts

The material that follows in based on a paper in a mathematical journal and a website [1-2].

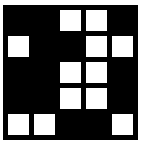
A 5×5 pantactic square is a 5×5 two-color grid pattern in which every 2×2 subpattern is different.

There are $2^4 = 16$ 2×2 patterns:



The identifying labels are obtained by following the rows left to right, taking white cells to be 0 and black cells to be 1. The resulting binary number is converted to hexadecimal.

Here is an example of a 5×5 pantactic square:



The 2×2 subpatterns from left to right, top to bottom are: DB247E81FA05C936. This string uniquely identifies this pantactic square.

Although there are $2^{25} = 33{,}554{,}432$ 5×5 two-color grid patterns, there are only 800 essentially different 5×5 pantactic squares. These 800 squares can be grouped into 16 categories according to common structural properties.

Properties of 5×5 Pantactic Squares

5×5 pantactic squares have some surprising properties, especially lack of symmetry. While many patterns are the same after some kind of rotation or refection, 5×5 pantactic squares are not: No combination of rotations, reflection, (horizontal, vertical, or diagonal), or color inversion of a 5×5 pantactic square produces the same 5×5 pantactic square.

Another property of 5×5 pantactic squares is a limitation of *connected paths* they can contain. A connected path in a grid pattern is a sequence of cells of the same color, all of which share an edge. For example, in

the black cells form a connected path, while in

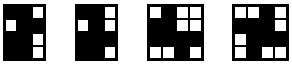
they do not, because in two places they connect only at corners.

A 5×5 pantactic square cannot have a connected path that reaches from one edge to the opposite one.

A *connected block* is a collection of cells of the same color in which the cells share edges. An example of a connected block is

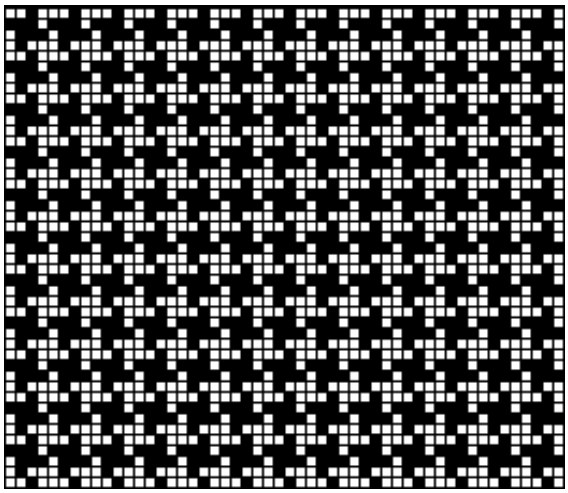
The maximum number of connected cells in a connected block in a 5×5 pantactic square is eight.

There are 50 essentially different *basic blocks* in 5×5 pantactic squares. Here are four of them:

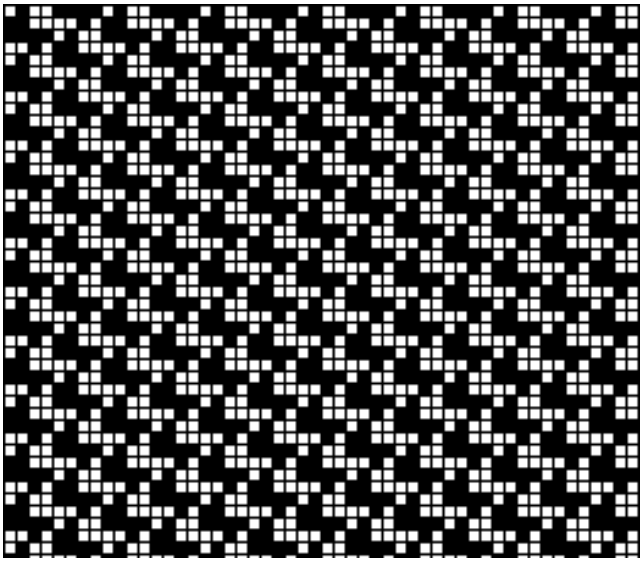


Pantactic Patterns

Some basic blocks can be arranged in ways that form repeating patterns of arbitrarily large size, all of whose 5×5 subpatterns are pantactic squares. Two examples are:



The arrangement here is a regular tiling.



The arrangement here is a tiling with a three-row offset.

Generalizations

There are other questions to be addressed. The literature on pantactic squares seems to be limited to 5×5 ones. Are there larger ones? Are there non-square pantactic designs? What about more than two colors?

These questions will be addressed in subsequent articles.

Reference

1. Astle, B. "Pantactic Squares", Mathematics Gazette 49 (1965), pp. 144-52.

2. Pantactic Squares
http://mcraeclan.com/mathhelp/CountingPantacticSquares.htm

Ralph E. Griswold
Department of Computer Science
The University of Arizona
Tucson, Arizona