Pattern Extension Schemata, Part 1: Replication and Inversion

The Replication Schema

Most weave structures are based on an interlacement motif that is repeated horizontally and vertically as necessary to form the complete pattern. See Figure 1.

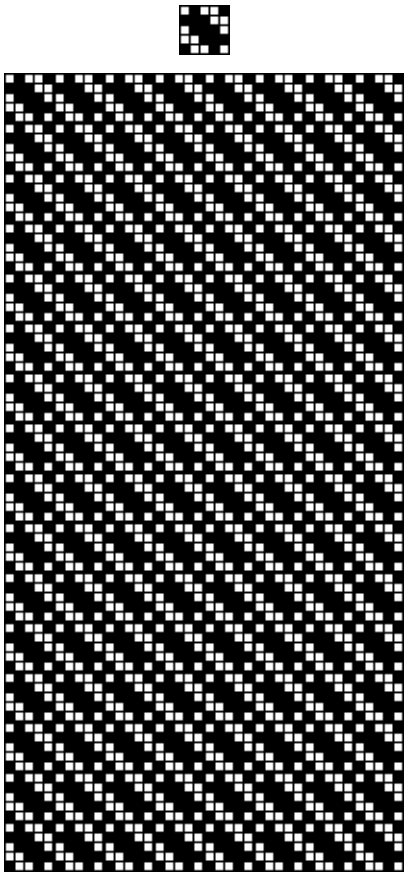

Figure 1. A Motif Repeated

This *repetition schema* is so common and fundamental that it often is taken for granted.

The Inversion Schema

There are other ways that a *foundation motif* can be used as the basis for larger patterns. One of the most interesting of these ways uses a motif and its inverse — the face and back of a pattern. In a drawdown, the inverse is obtained by changing black cells to white and white cells to black. See Figure 2.

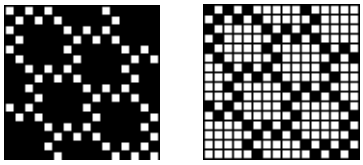

Figure 2. A Pattern and Its Inverse

The *inversion schema* juxtaposes a foundation motif and its inverse horizontally and then juxtaposes the result with *its* inverse vertically. See Figures 3 and 4.

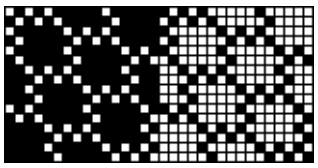

Figure 3. Step 1 in the Inversion Schema

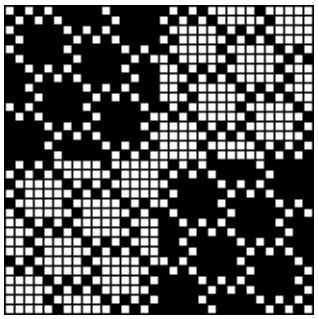

Figure 4. Step 2 in the Inversion Schema

This process is then iterated until a sufficiently large pattern is obtained. The second iteration in the inversion schema for the example above is shown in Figure 5.

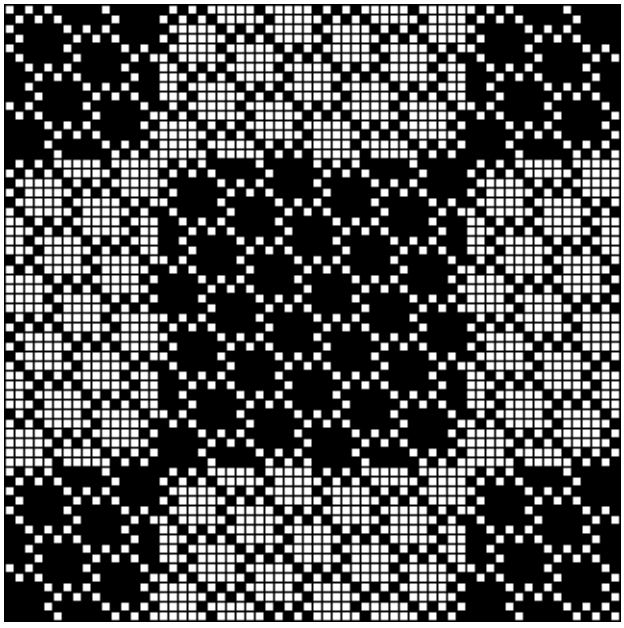

Figure 5. Iteration 2

The foundation motif for the inversion schema can be very small and still produce interesting results. For example, a single black cell (not an interlacement pattern *per se*) develops as shown in Figure 6.

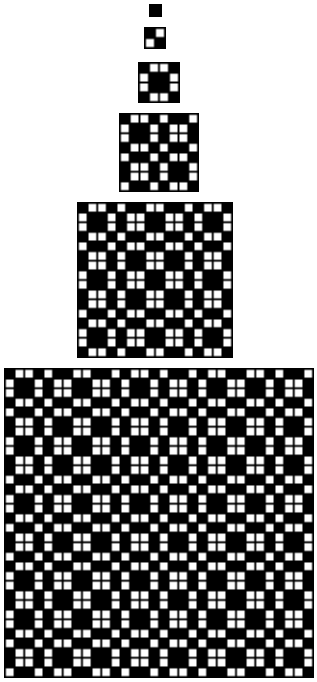

Figure 6. Development from a Single Cell

The iterations shown in Figure 6 represent stages in the development of the infinite Morse-Thue Carpet, a two-dimensional extension of the Morse-Thue sequence, one of the most important of all integer sequences [1].

Incidentally, if the fundamental motif is a single white cell, the result is the back of the Morse-Thue Carpet.

Patterns produced by the inversion schema have several interesting properties:

- Successive iterations of the inversion schema double the size of the pattern. This limits the number of iterations that are practical to use, and in any iteration the size of the pattern may jump from smaller than the desired overall size to much larger than the desired overall size. An overly-large pattern can, of course, be trimmed.
- Successive iterations of the inversion schema appear to produce periodic patterns but the patterns are not periodic. That is, there is no smaller motif that can produce them using the repetition schema. The appearance of periodicity is visually attractive, but the departure from it adds interest — even if the reason is not readily evident.
- Patterns from the inversion schema are, in fact, finite portions of infinite fractal patterns. Such patterns are self similar and an infinite pattern contains infinitely many copies of itself within itself.
- The loom resources required for weaving patterns from the inversion schema (shafts and treadles, for example) are the same for all iterations from the first on. The Morse-Thue carpet, for example, requires only two shafts and two treadles, regardless of the number of iterations. The reason for this is fundamentally the same as the reason why large patterns formed by the repetition schema require no more loom resources than the fundamental motif does: All the structural variety exists in the first iteration of the inversion schema; beyond that, the complexity of the pattern is obtained by the varying threading and treadling sequences.
- The upper-left three-quarters of any iteration of the inversion schema past the first, when used as the fundamental mo-

tif for the repetition schema, matches the entire inversion schema pattern. That is, extension by repetition from 3/4ths to the dimensions of the full inversion schema pattern matches the inversion schema pattern exactly. See Figures 7-9. Therefore, the inversion schema can be carried out until a pattern of the desired appearance is reached and then continued by the replication schema. The result of repeating the 3/4ths motif of an inversion pattern is, of course, different from the next inversion iteration, See Figures 10 and 11 on the next two pages. Switching from inversion to repetition is useful because repetition is easier to do than inversion and size is more readily controlled.

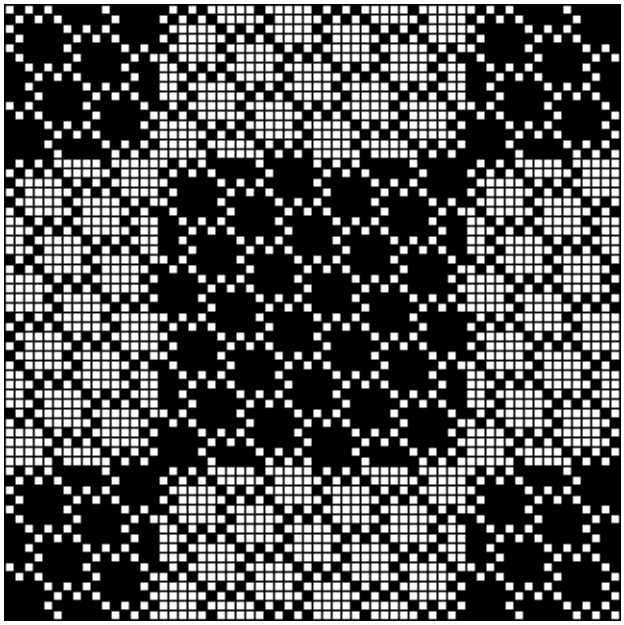

Figure 7. Second Iteration (As Figure 5)

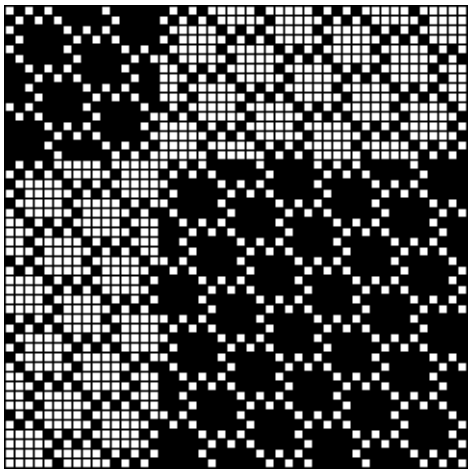

Figure 8. Upper-Left 3/4ths of Figure 7.

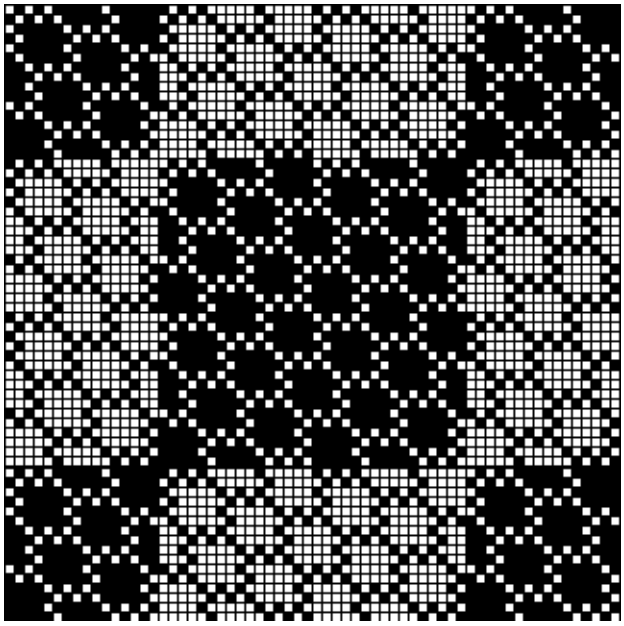

Figure 9. Extension by Repetition of Figure 8 to Size of Figure 7.

The Appendix shows the development of patterns from several foundation patterns.

Relation of Inversion to Combination Weaves

Some combination weaves use combinations of the face and back of a motif as the basis

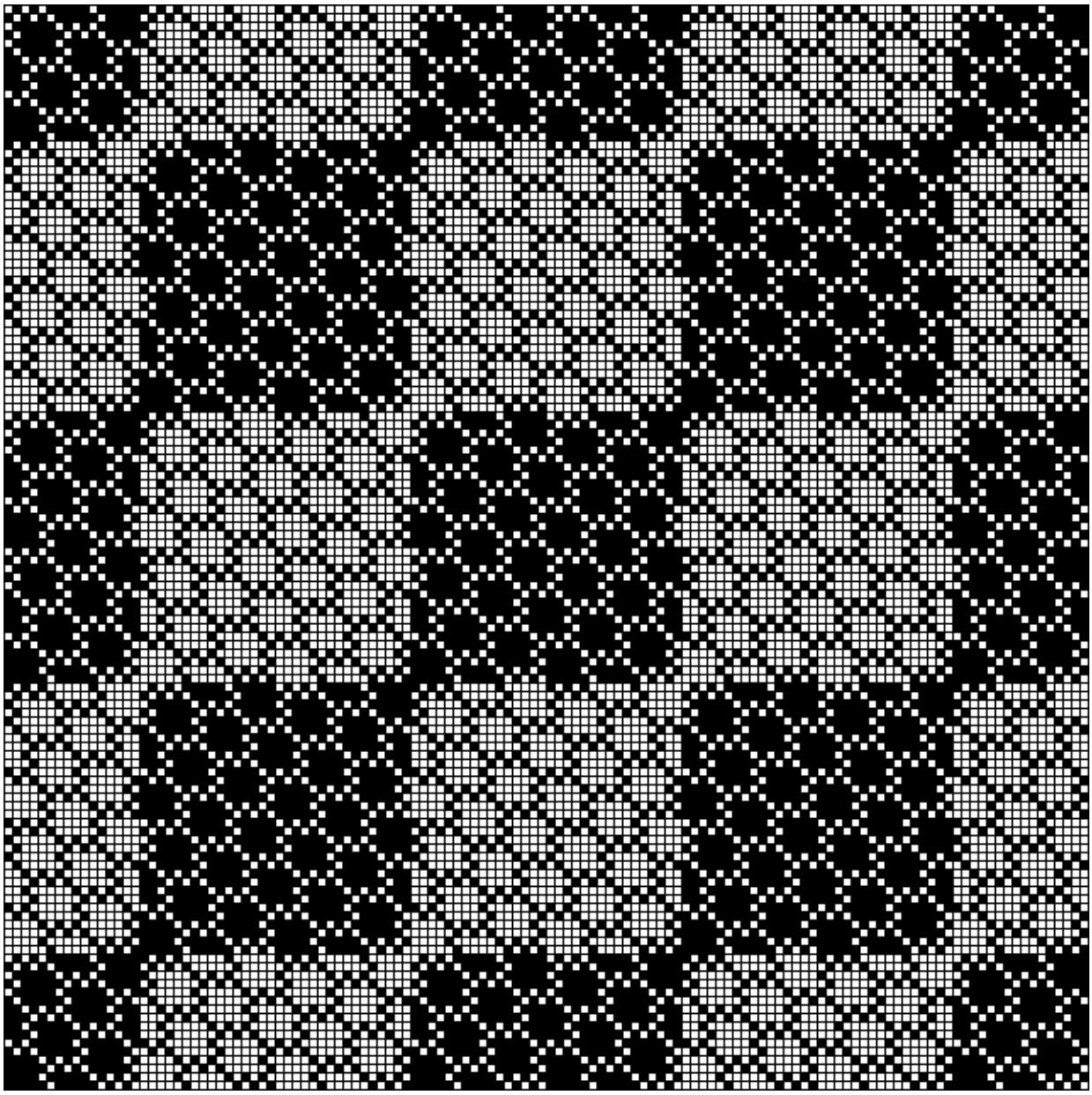

Figure 10. Repetition of the 3/4ths Motif

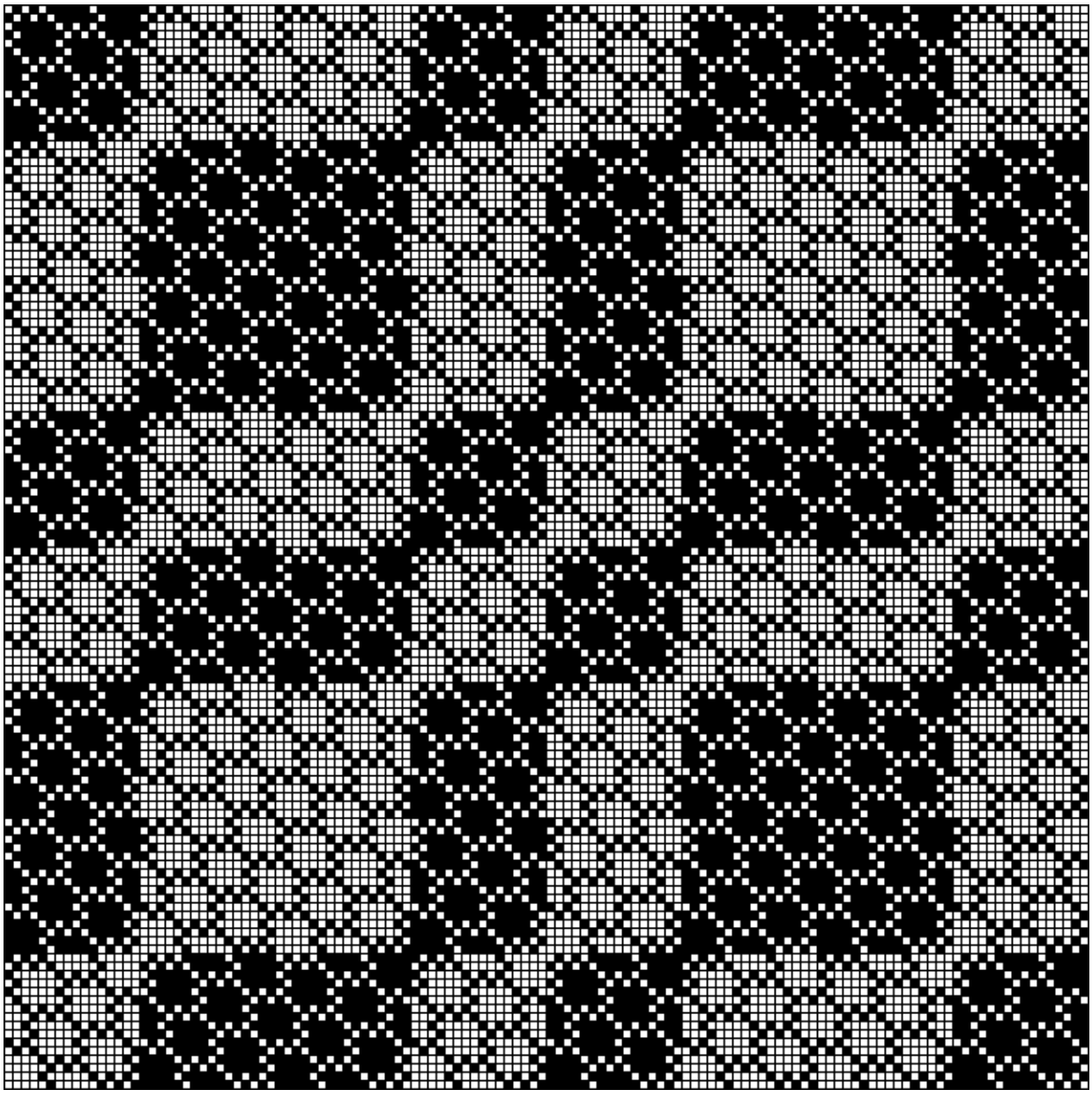

Figure 11. The Next Inversion Iteration

for developing a larger motif [2,3]. Many checkerboard patterns are based on this idea. Figure 12 shows an example.

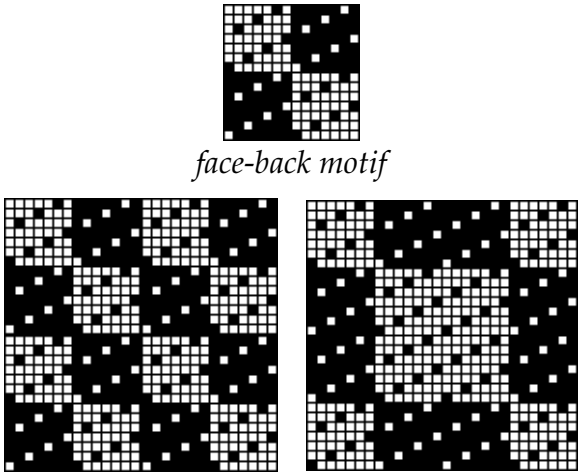

Figure 12. Replication Versus Inversion

The difference between this method and the inversion schemata is that the face-back combination is used only once to form a motif that is then repeated. Compare Figures 10 and 11 on this basis.

Other Pattern Extension Schemata

There are many possible schemata that can be used to produce interesting patterns. Obvious examples are geometric transformations, such as successive rotations.

Different schemata also can be used in successive iterations.

Some of these ideas will be explored in more detail in future articles on the subject.

References

1. *The Morse-Thue Sequence*, Ralph E. Griswold, 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_mt.pdf
2. *A Handbook of Weaves,* G. H. Oelsner, 1915, Dover Reprint, 1952.
3. *Textile Designing*, C. J. Brickett, International Textbook, 1924.

Ralph E. Griswold
Department of Computer Science
The University of Arizona
Tucson, Arizona

Appendix: Examples of Inversion Schema Development