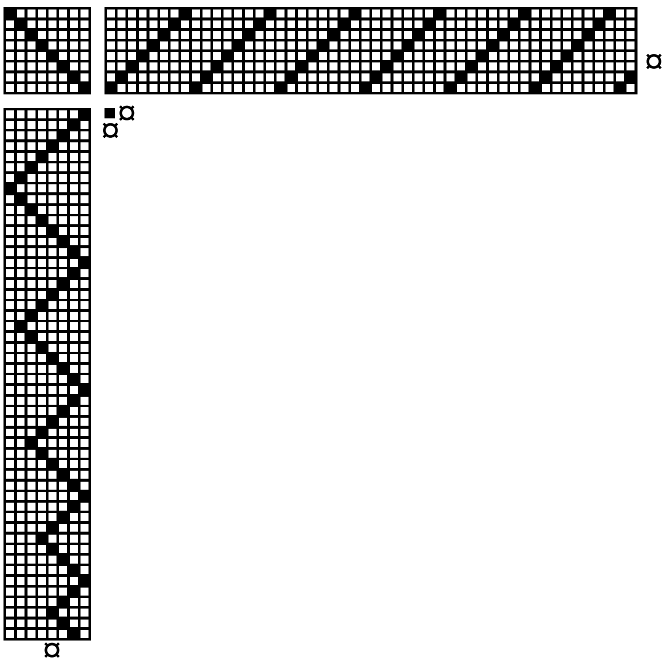
Drafting with Sequences

Shafts and treadles in drafts are numbered for identification. The numbers of the shafts through which successive warp threads pass form a sequence, as do the numbers of the treadles for successive picks. Consider the following draft, in which the arrows indicate the orientation:



The threading is an upward straight draw. The sequence is:

1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2

The treadling sequence is more complicated:

1, 2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 7, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, 3, 4, 3, 2

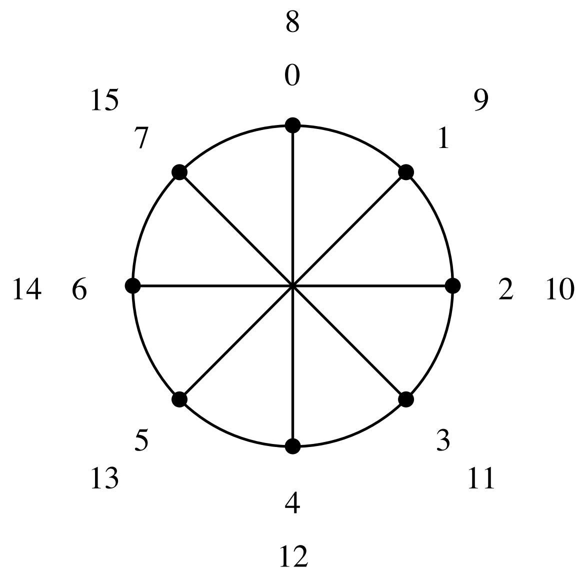
These two sequences, in combination with the tie-up, define the structure of the weave.

Threading and treadling sequences often have distinctive patterns, as in the repeat for the threading sequence above. In the case of a repeat, it's only necessary to know the basic unit, which we'll indicate by brackets:

[1, 2, 3, 4, 5, 6, 7, 8]

Modular Arithmetic

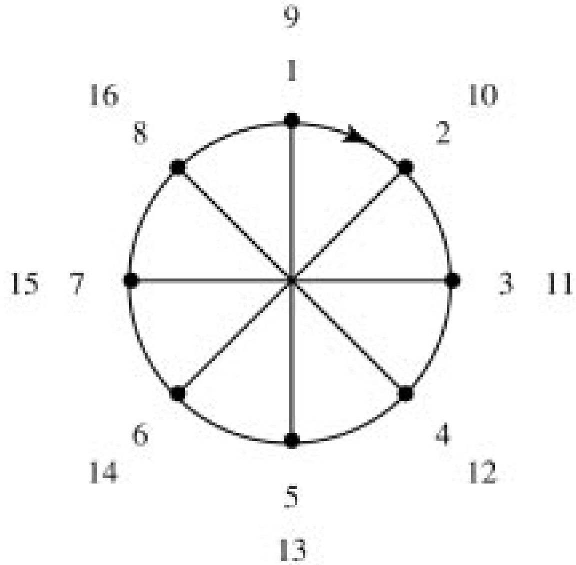
Since looms have a fixed number of shafts and treadles, the sequences usually are most easily understood in terms of modular arithmetic, sometimes called clock or wheel arithmetic, in which numbers go around a circle clockwise, starting with 0. If there are 8 shafts, there are 8 equally spaced points on the circle from 0 to 7:



The numbers on the inner circle are those that exist in the modular arithmetic. If we continue beyond 7, as shown in the outer ring, the numbers wrap around the wheel. Numbers on the same spoke are equivalent. For example, 0 and 8 are equivalent, 1 and 9 are equivalent, 2 and 10 are equivalent, and so on. Another way to look at it is that when 9 is introduced into modular arithmetic with 8 shafts, it *becomes* 1, and so on.

Shaft Arithmetic

Although modular arithmetic uses the number 0 as a starting point, most persons count from 1. We'll keep this convention, which is used for numbering shafts and treadles. This is easily accomplished by rotating the wheel counterclockwise by one position:



Notice that 1 and 9 are still equivalent, as are 2 and 10, and so on. If there are 8 shafts, there are 8 positive numbers. 0 has gone away, but it will be back.

For sequences, shafts and treadles are handled the same way, so we'll call this *shaft arithmetic*, with the understanding that it applies to treadles also. Of course, most facts about shaft arithmetic hold for ordinary modular arithmetic.

In shaft arithmetic, an upward straight draw for 8 shafts is described by the positive integers in sequence:

1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, …

and wrapped around the shaft circle to produce

1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 4, 5, 6, 7, 8, …

The point is that an upward straight draw comes from the most fundamental of all integer sequences, the positive integers in order. (We'll discuss downward straight draws later.)

Drafting with Sequences

The idea behind drafting with sequences is that many sequences have interesting patterns, which often become more interesting in shaft arithmetic. In fact, many sequences show repeats when cast in shaft arithmetic. When this is the case, the entire sequence can be represented by the repeat. For example, the shaft sequence for an upward straight draw for 8 and 10 shafts are represented by

[1, 2, 3, 4, 5, 6, 7, 8]

and

[1, 2, 3, 4, 5, 6, 7, 8, 9, 10]

respectively.

Note: Not all sequences produce repeats in shaft arithmetic. For example, the prime numbers, which are divisible only by 1 and themselves, do not show a repeat in shaft arithmetic (or in any other arithmetic).

Patterns in Sequences

Sequences may produce interesting woven patterns when they are used for threading and treadling.

There are many, many well-known integer sequences. The Fibonacci sequence, which has many connections in nature, design and mathematics, is one of the best known and most thoroughly studied of all integer sequences. The Fibonacci sequence starts with 1 and 1. Then each successive number (*term*) is the sum of the preceding two:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,…

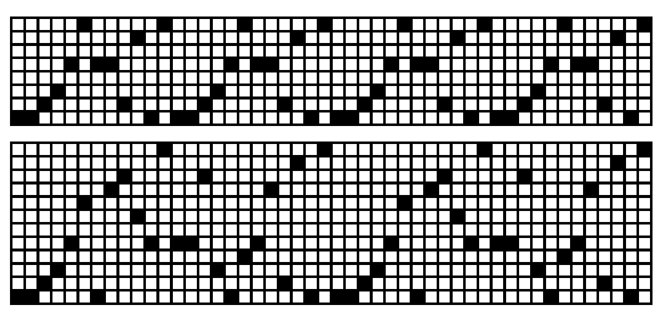
As the sequence continues, the numbers get very large. For example, the 50th term in the Fibonacci sequence is more than 12 billion. Shaft arithmetic brings this sequence under control. For 8 shafts, the result is

1, 1, 2, 3, 5, 8, 5, 5, 2, 7, 1, 8, 1, 1, 2, 3, 5, 8, 5, 5, 2, 7,
1, 8, 1, 1, 2, 3, 5, 8, 5, 5, 2, 7, 1, 8, …

As you can see, there is a repeat, so the entire sequence can be represented by

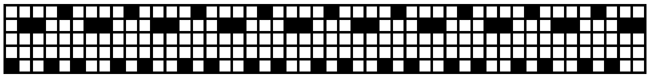
[1, 1, 2, 3, 5, 8, 5, 5, 2, 7, 1, 8]

Patterns in sequences are more easily seen if they are plotted, as in the grids used in weaving drafts. For 8 and 12 shafts, the Fibonacci sequence looks like this:

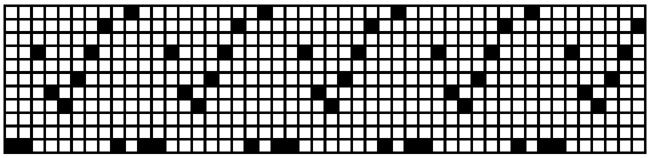


Here are some other simple sequences and what they look like for various numbers of shafts.

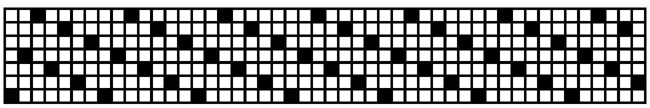
The squares for 5 shafts:



The cubes of the Fibonacci numbers for 11 shafts:

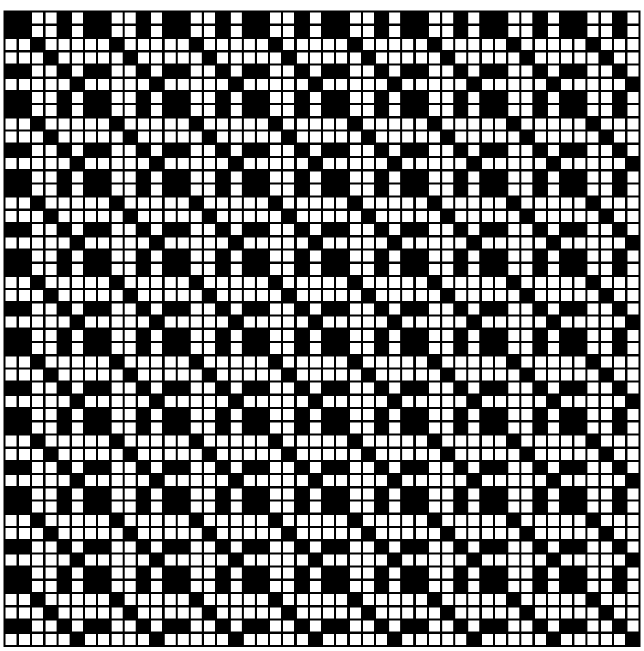


Every third positive integer for 7 shafts:

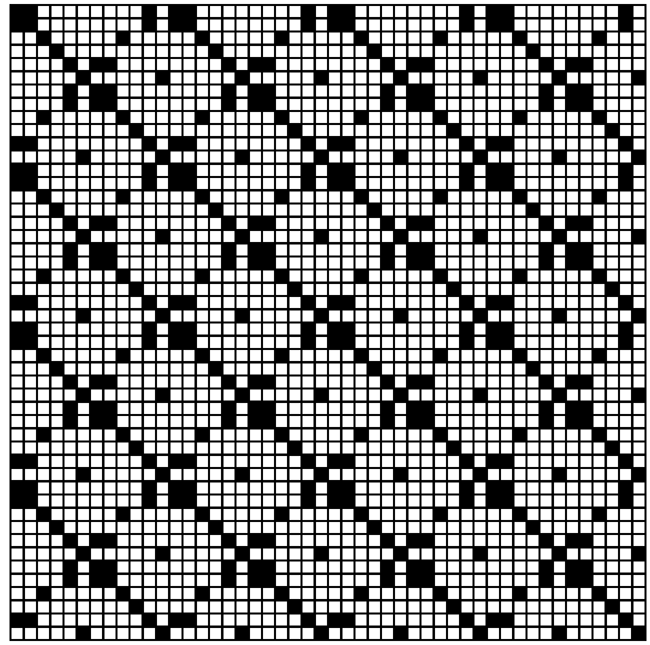


The patterns such sequences produce in weaves depend on many factors. To keep things simple to begin with, we'll use direct tie-ups and treadling as drawn in (that is, the same sequence for the threading and the treadling). Even in this very limited framework, interesting woven patterns abound.

Here is a drawdown for a few repeats of the Fibonacci sequence for 4 shafts.



The pattern looks quite different for 8 shafts, although you'll notice structures in common:

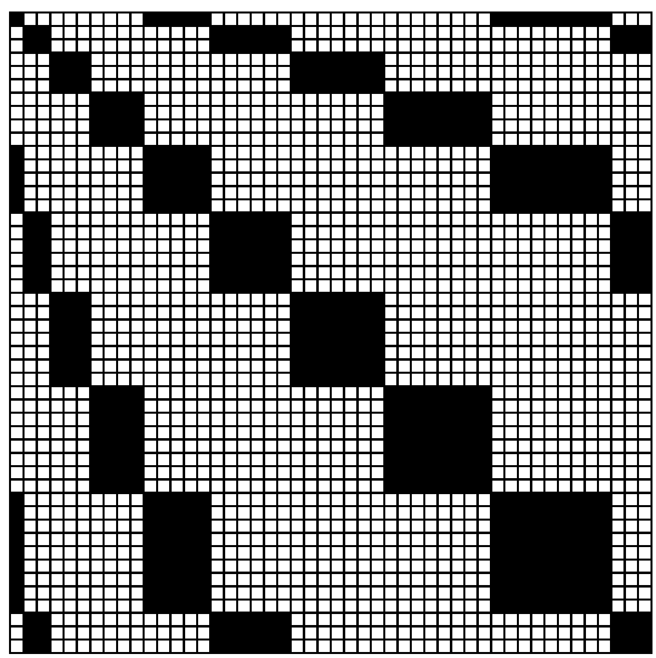


A simple sequence that produces interesting patterns is the "multi" sequence, which starts with a single 1 and is followed by 2 copies of 2, 3 copies of 3, and so on:

1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, …

Note that there are no repeats in shaft arithmetic for this sequence, since the "width" of the repeated integer blocks constantly increases.

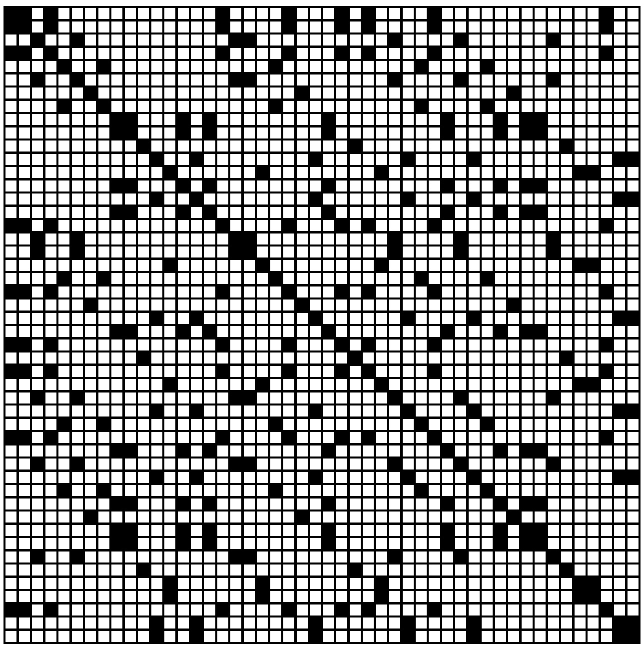
The drawdown for the multi sequence for 4 shafts is:



One way to produce interesting sequences is to combine other sequences, such as interleaving the terms of two sequences. For example, interleaving the positive integers and the Fibonacci sequence produces

1, 1, 2, 1, 3, 2, 4, 3, 5, 5, 6, 8, 7, 5, 8, 5, 1, 2, 2, 7, 3, 1, 4, 8, 5, 1, 6, 1, 7, 2, 8, 3, 1, 5, 2, 8, 3, 5, 4, 5, 5, 2, 6, 7, 7, 1, 8, 8 …

The drawdown for 8 shafts is:



Other tie-ups and threading sequences and treadling sequences that are different produce all kinds of interesting results.

Creating interesting weaves by drafting with sequences requires judicious selection and combination of sequences, the number of shafts and treadles, and tie-ups. An understanding of the properties of the sequences used may help, but a little luck and some experimentation also can lead to pleasant surprises. The process is a nice combination of artistic sense, creative talent, a modicum of arithmetic, and finding the hidden structures that abound in integer sequences.

Finding Interesting Integer Sequences

Interesting integer sequences can come from many sources. It helps if you have a computer with a program that can do simple arithmetic so that you can invent your own. There also are many on-line sources of sequences. By far the most extensive one is the "Encyclopedia of Integer Sequences" (EIS) [1]

Beware, though — this site contains a lot of esoteric mathematical material and its vastness can be overwhelming. It's like a "Haystack from Hell", but the needles to be found within are made of precious metals.

Getting Shaft Sequences

There are shaft sequences for a few integer sequences and various numbers of shafts on Reference 2.

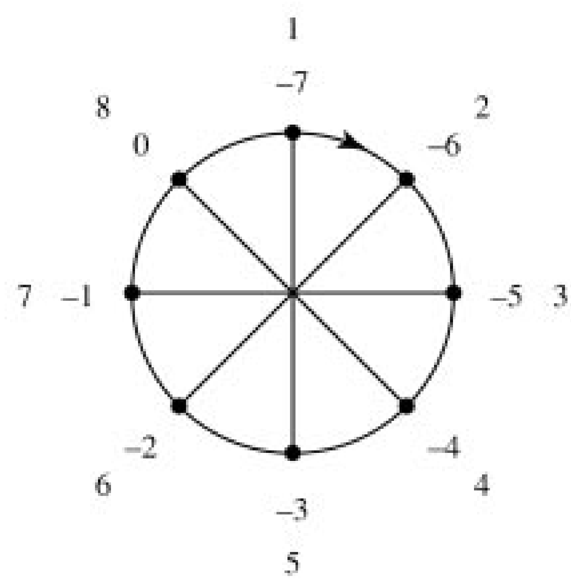
These sequences provide an easy way to start, but you'll want more if you decide you're really interested in drafting with sequences.

You can find many integer sequences ready made, but in order to do your own drafting, you need to be able to convert them to shaft sequences for different numbers of shafts. The method is simple: Divide each term by the number of shafts and take the reminder. For example, for 8 shafts, the reminder of 13 divided by 8 is 5, which is the shaft number for 13. That gives you the corresponding term in the shaft sequence. It helps if you have a program or calculator that can do integer arithmetic and produce remainders.

There's one more complication — 0 and negative numbers. The way to deal with these is indicated by looking at what happe□ns when you have negative integers in increasing sequence as they cross over to the positive integers:

…, –7, –6, –5, –5, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, …

Now think of the modular wheel and what happens if you wrap this sequence of numbers around it. For 8 shafts, it looks like this:



In other words, –1 becomes 7, –2 becomes 6, and so on. Note that 0, which we've been hiding, becomes 8.

Perhaps you now see the integer sequence that produces a downward straight draw:

0, –1, –2, –3, –4, –5, –6, –7, –8, –9, –10, …

All that's needed to convert a non-positive remainder to a shaft number is to add it to the number of shafts. For –1, for example,

$8 + (-1) = 7$

Despite this long-winded discussion, getting shaft sequences from integer sequences is not difficult at all.

The interesting part remains — trying it and designing drafts.

References

1. Encyclopedia of Integer Sequences:
http://www.research.att.com/~njas/sequences/

2. Integer Sequences:
http://www.cs.arizona.edu/patterns/weaving/sequences.html

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