Patterns from Term-Replication Sequences

You probably have seen patterns like the one in Figure 1 and its mirrored extension in Figure 2.

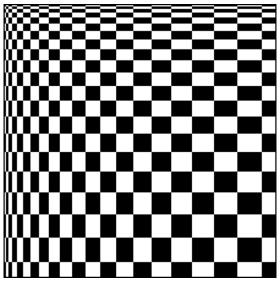

Figure 1. Pattern

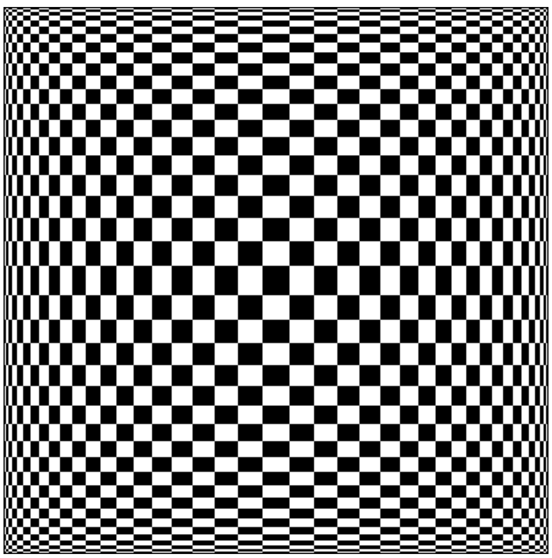

Figure 2. Mirrored Pattern

These patterns come from a very simple sequence:

1 2 2 3 3 3 4 4 4 4 5 5 5 5 5 …

For example, if this sequence is used for the threading and treadling sequences in a weaving draft with a tabby tie-up, the resulting drawdown is as shown in Figure 1.

In the sequence above, each term is replicated according to its value. We know of no good name for this sequence. We have called it the multi sequence in previous articles [1] and the On-Line Encyclopedia of Integer Sequences [2] refers to it as "*n* appears *n* times", which is descriptive but far from elegant.

The sequence above is one of a class of sequences obtained by applying *term replication functions* to *bases sequences*.

For the example above, the base sequence is the positive integers, $I^+ = 1\ 2\ 3\ 4\ 5\ \ldots$ and the replication function is $\mathrm{r}(v) = v$, where v is the value of the term.

If the base sequence is the Fibonacci numbers, $F = 1\ 1\ 2\ 3\ 5\ 8\ \ldots$, then this rule yields

1 1 2 2 3 3 3 3 5 5 5 5 5 5 8 8 8 8 8 8 8 8 …

Compact Representations of Term Replication

Sequences in which terms are replicated may be difficult to understand if terms are written out in the usual fashion.

One way to reduce visual clutter is to list replicated terms only once along with their replication factor. We'll use the notation

$$\underline{i}_j$$

to indicate that there are j copies of i. Thus, the result of applying $\mathrm{r}(v) = v$ to I^+ and the primes, $P = 2\ 3\ 5\ 7\ \ldots$, can be written as

$$1\ \underline{2}_2\ \underline{3}_3\ \underline{4}_4\ \underline{5}_5\ \ldots$$
$$\underline{2}_2\ \underline{3}_3\ \underline{5}_5\ \underline{7}_7\ \ldots$$

We use 1 rather than $\underline{1}_1$, and similarly for other non-replicated terms, to further reduce visual clutter.

Another way to represent the results of applying a replication function to a base sequence is to write the base sequence above the replication sequence, with a bar separating the two. For the examples above, the representations are

$$\frac{1\ 2\ 3\ 4\ 5\ \ldots}{1\ 2\ 3\ 4\ 5\ \ldots}$$

$$\frac{2\ 3\ 5\ 7\ \ldots}{2\ 3\ 5\ 7\ \ldots}$$

For named sequences, a simpler, linear typographical form can be used, as in P/F.

Value-Based Replication Functions

Replication functions whose values are determined solely by term values are called value-based.

Many kinds of value-based replication functions are possible, such as the following:

$$\mathrm{r}(v) = 1 \qquad [1]$$

$$\mathrm{r}(v) = v \qquad [2]$$

$$\mathrm{r}(v) = v + 2 \qquad [3]$$

$$\mathrm{r}(v) = v \text{ smod } 5 \qquad [4]$$

$$\mathrm{r}(v) = \begin{cases} 1 & v \text{ even} \\ 2 & v \text{ odd} \end{cases} \qquad [5]$$

$$\mathrm{r}(v) = \begin{cases} 1 & v \text{ even} \\ 0 & v \text{ odd} \end{cases} \qquad [6]$$

Eqn. 1 leaves the base sequence unchanged. Eqn. 2 produces the results described previously. Eqn. 3 is like Eqn. 2 except that 2 replications are added. In Eqn. 4, the replication factor is reduced shaft-modulo 5 [1], so that values whose residues are 1 smod 5 are not replicated, values whose residues are 2 smod 5 are replicated two times, and so on.

In Eqns. 5 and 6, the result depends on the parity of the value. In Eqn. 5 even values are not replicated, while odd ones are duplicated. In Eqn. 6, even values are not replicated and odd values are discarded (being replicated 0 times).

Note that in Eqns. 2 and 3, replication factors increase without limit as v does. In the other equations, the replication factors are bounded regardless of how large v is.

Position-Based Replication Functions

Replication factors can be based on the positions of terms instead of their values, position being the number of the term in the sequence. For example, in P, 2 is term 1, 3 is term 2, 5 is term 3, 7 is term 4, and so on.

For example, if p is the position of a term in a sequence, the replication function

$$\mathrm{r}(p) = \begin{cases} 1 & p \text{ odd} \\ 2 & p \text{ even} \end{cases} \qquad [7]$$

doubles even-numbered terms but not the odd-numbered terms.

The replication function

$$\mathrm{r}(p) = p \qquad [8]$$

replicates by the position of the term. For I^+, Eqn. 8 produces the same results as Eqn. 2. For P, it produces

$$2\ \underline{3}_2\ \underline{5}_3\ \underline{7}_5 \ldots$$

Value- and Position-Based Replication Functions

Replication functions can depend both on value and position. An example is

$$\mathrm{r}(v, p) = \begin{cases} v & p \text{ odd} \\ p & p \text{ even} \end{cases} \qquad [9]$$

For F, Eqn. 9 produces

$$\underline{1}_3\ \underline{2}_2\ \underline{3}_3\ \underline{5}_5\ \underline{8}_6 \ldots$$

Replication Sequences

Replication factors can be determined independently of the base sequence. For example, for the base sequence I^+ and the replication sequence P, we have

$$I^+ / P =$$

$$\frac{1\ 2\ 3\ 4\ \ldots}{2\ 3\ 5\ 7\ \ldots} =$$

$$\underline{1}_2\ \underline{2}_3\ \underline{3}_5\ \underline{4}_{7\ \ldots} =$$

$$1\ 1\ 2\ 2\ 2\ 3\ 3\ 3\ 3\ 3\ 4\ 4\ 4\ 4\ 4\ 4\ 4\ \ldots$$

As another example, consider the 1-based Morse-Thue sequence [3], $M = 1\ \ 2\ \ 2\ \ 1\ \ 2\ \ 1\ \ 1\ \ 2 \ldots$ as the base sequence and I^+ as the replication sequence:

$$M\ / I^+ =$$

$$\frac{1\ \ 2\ \ 2\ \ 1\ \ 2}{1\ \ 2\ \ 3\ \ 4\ \ 5}\ \begin{matrix}1\ \ 1\ \ 2 \ldots \\ .6.\ \ 7\ \ 8\end{matrix} =$$

$$1\ \underline{2}_5\ \underline{1}_4\ \underline{2}_5\ \underline{1}_{13}\ \underline{2}_8 \ldots =$$

$$12\ \ 2\ \ 2\ \ 2\ \ 2\ \ 1\ \ 1\ \ 1\ \ 1\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ \ \ 1\ \ 1\ \ 1$$
$$1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2\ \ 2 \ldots$$

Term-Replication Sequence Patterns

Patterns derived from term-replication sequences may not be suitable, as-is, for interlacement patterns in weaving for structural reasons. Such patterns, however, may make good block patterns for profile drafting.

As in all such things, designing good patterns based on term replication requires a combination of experience, skill, and creativity.

The Appendix A shows some examples that can be used as a basis for experimentation. Appendix B shows some examples of mirrored patterns based on term replication sequences.

All the examples in the appendices are produced using tabby tie-ups with treadling as drawn in. Hint, hint … .

References

1. Ralph E. Griswold, "Drafting with Sequences", 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/reg_seqd.pdf

2. On-Line Encyclopedia of Integer Sequences:
http://www.research.att.com/~njas/sequences/index.html

3. Ralph E. Griswold, "The Morse-Thue Sequence", 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_mt.pdf

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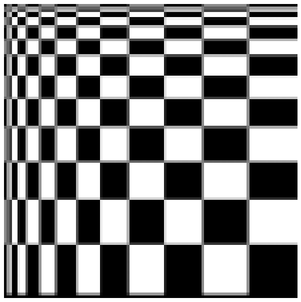
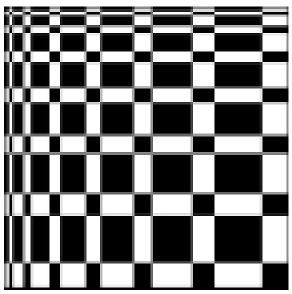
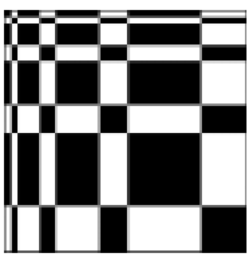
Appendix A — Patterns Derived from Term-Replication Sequences



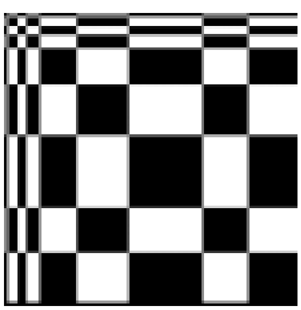
I^{+} / P



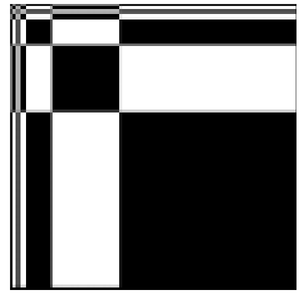
F / I^{+}



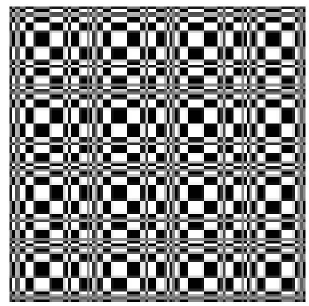
F / P



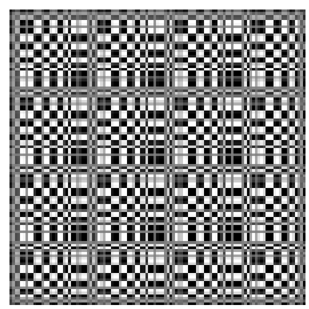
M / P



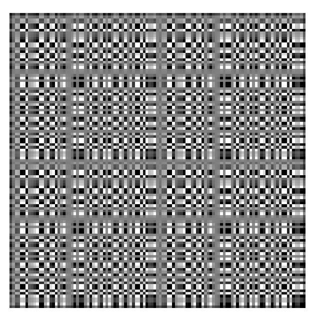
M / F



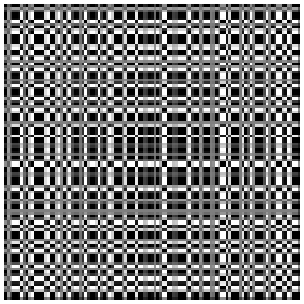
M / (F **smod 7)**



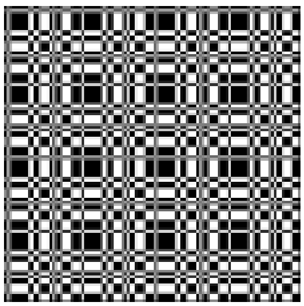
I / (F **smod 7)**



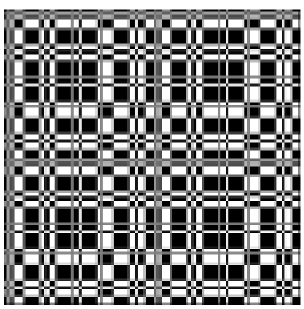
I / (F **smod 5)**



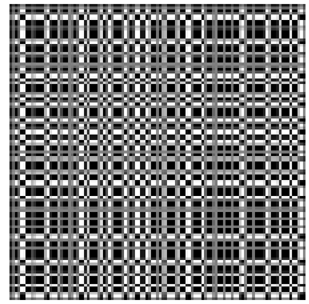
***F* / (*F* smod 5)**



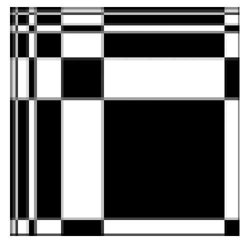
(*F* smod 5) / (*F* smod 5)



(*F* smod 3) / (*F* smod 5)

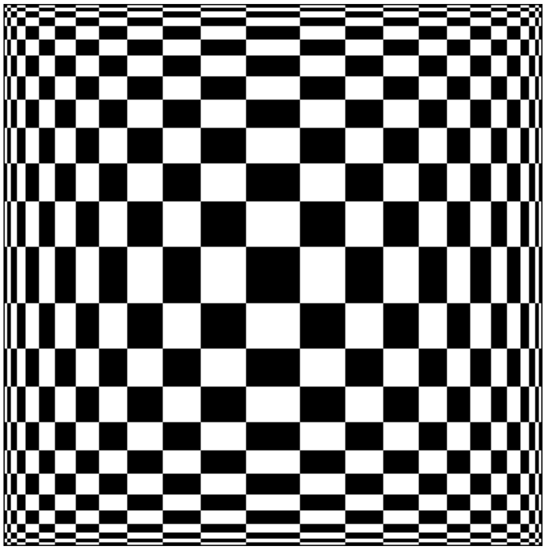


(I^+ smod 3) / (*F* smod 5)

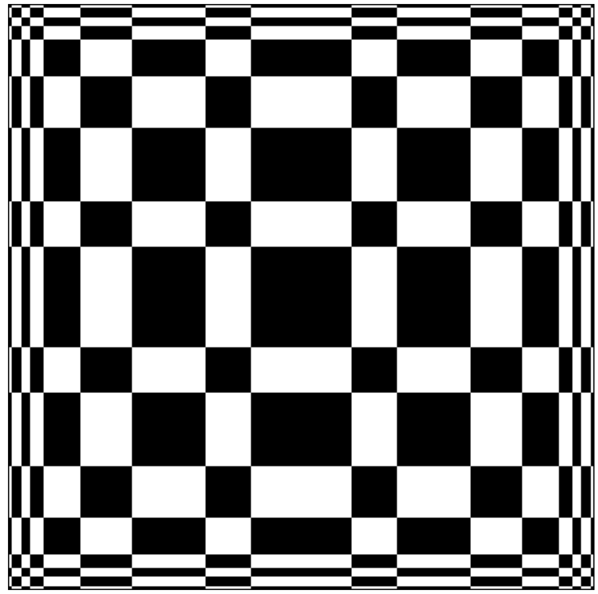


Eqn. 8 Applied to *F*

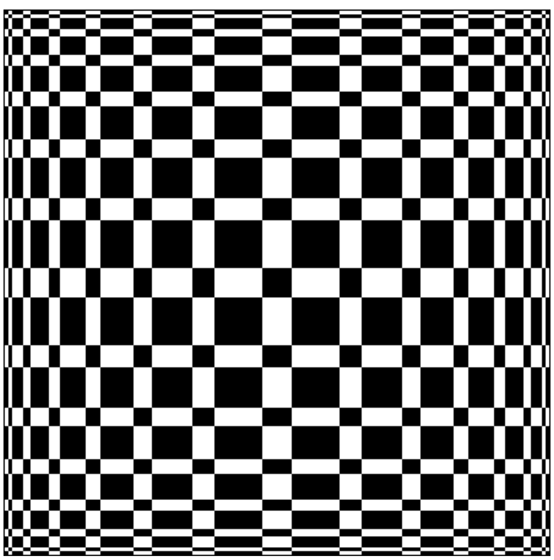
Appendix B — Mirrored Patterns Derived from Term-Replication Sequences



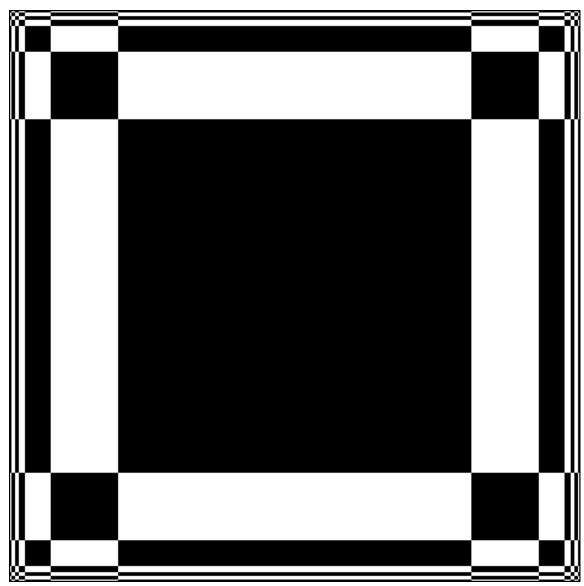
I^{+}/P



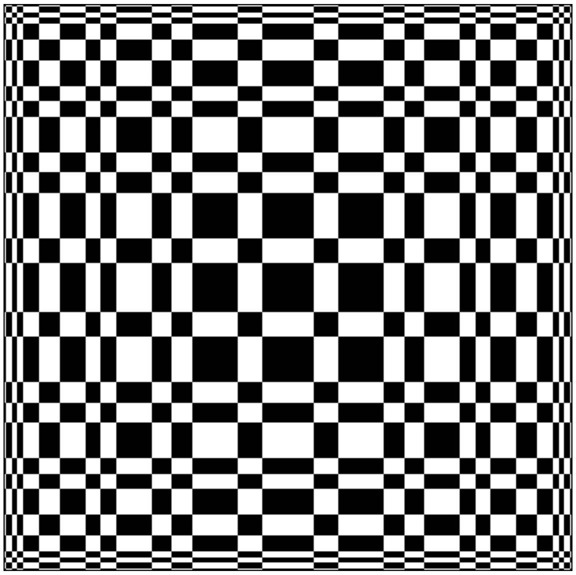
M/P



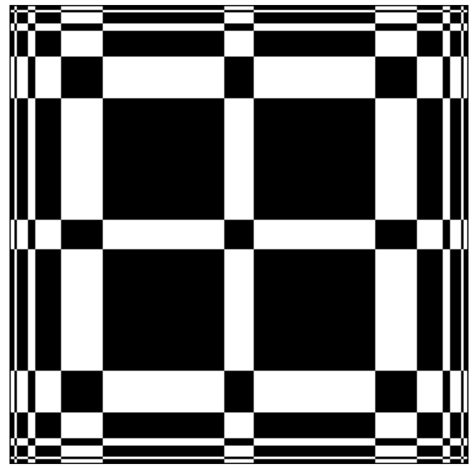
F/I^{+}



M/F



M* / *I



Eqn. 8 Applied to *F*