T-Sequences, Part 2: Extension

The first article in this series [1] introduced the concept of t-sequences: sequences of integers that can be used for threading and treadling. This article, describes ways in which t-sequences can be extended.

Concatenation

The most fundamental operation on t-sequences is appending one to another to form a longer one. This is called *concatenation*.

Concatenation of t-sequences is denoted by

$$S \mid T$$

in which the result is a new sequence consisting of the terms of S followed by the terms of T.

For example if

$$S = [1, 2, 3, 4, 5, 6, 5, 4, 3, 2]$$

and

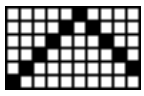
$$T = [1, 2, 3, 2, 1, 2, 3, 4, 5, 6, 7, 8]$$

then

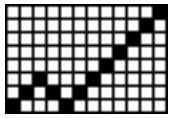
$$S \mid T = [1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, 3, 2, 1, 2, 3, 4, 5, 6, 7, 8]$$

Here is the graphic representation:

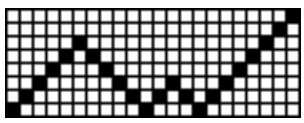
S



T



$S \mid T$



The empty sequence Θ is the identity with respect to concatenation. That is,

$$(S \mid \Theta) = (\Theta \mid S) = S$$

for all S.

Often many t-sequences are concatenated, one after the other. To handle such cases conveniently, the notation

$$\mid (S_1, S_2, \ldots, S_n)$$

denotes the concatenation of $S_1, S_2, \ldots, S_n$.

Repetition

Repetition is one of the most common operations on t-sequences. Repetition consists of concatenating a sequence with itself, perhaps several times.

Repetition is denoted by

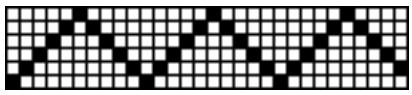
$$S \times i$$

where i, an integer ≥ 0, specifies the number of repetitions.

For example, if S is as given in the preceding section, then

$$(S \times 3) = [1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2]$$

Here is what it looks like as a grid plot:



$(S \times 1) = S$ and $(S \times 0) = \Theta$, the empty sequence for all S.

Extension

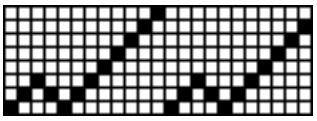
It sometimes is desirable to repeat a sequence to a specific length that is not an even multiple of the length of the sequence.

This operation is called *extension* and is denoted by $S \Rightarrow i$, where $i \geq 0$ is the length of the new sequence.

For example, if T is as given previously,

$$(T \Rightarrow 23) = [1, 2, 3, 2, 1, 2, 3, 4, 5, 6, 7, 8, 1, 2, 3, 2, 1, 2, 3, 4, 5, 6, 7]$$

Here is what it looks like as a grid plot:



The extension length i may be less than $\lambda(S)$, in which case truncation at the right occurs. For example,

$$(T \Rightarrow 9) = [1, 2, 3, 4, 5, 6, 5, 4, 3]$$

Of course, $(S \Rightarrow 0) = \Theta$ for all S.

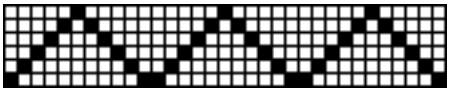
Duplicate Terms

Although concatenation and its two specialized forms, repetition and extension, are simple and fundamental operations, problems may arise if the last term in a sequence is the same as the first term in the sequence appended to it. Such duplicate terms may appear as undesirable artifacts of the concatenation and in some weaving contexts may cause structural problems.

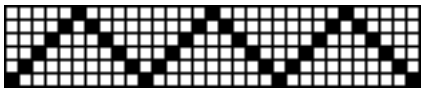
For example, if

$$S = [1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1]$$

and duplicates at the boundaries of concatenation are not removed, $S \times 3$ would be as shown as:



If duplicate terms at the boundaries of concatenation are removed, however, the result is as shown here:



Whether or not duplicates that result from concatenation should be removed is a matter of context and not a property of the sequences involved. More often than not, duplicate removal is desired, so the operations of concatenation, repetition, and extension do that.

There are alternative versions of these operations that do not remove duplicates. These are denoted by $S \mid_+ T$, $S \times_+ i$, and $S \Rightarrow_+ i$. For example, for the sequence S given above,

$$(S \times_+ 3) = [1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1]$$

Note: Any duplicates *within* a sequence are unaffected by any of the concatenation operations.

Summary

With duplicate removal:

$S \mid T$	concatenation
$\mid(S_1, S_2, \ldots, S_n)$	concatenation
$S \times i$	repetition
$S \Rightarrow i$	extension

Without duplicate removal:

$S \mid_+ T$	concatenation
$\mid_+(S_1, S_2, \ldots, S_n)$	concatenation
$S \times_+ i$	repetition
$S \Rightarrow_+ i$	extension

Reference

1. Ralph E. Griswold, "T-Sequences, Part 1: Introduction", 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_ts01.pdf

Ralph E. Griswold
Department of Computer Science
The University of Arizona
Tucson, Arizona