T-Sequences, Part 4: Symmetries

Symmetry is one of the most powerful tools for producing aesthetically pleasing patterns. In t-sequences, the main use of symmetry is in concatenating a sequence and its reversal to produce a palindrome. Geometrically, reversal is horizontal reflection.

Horizontal Reflection

Horizontal reflection reverses the order of the terms in a sequence left to right. Horizontal reflection is denoted by $\leftrightarrow S$. For example, if

$$S = [2, 4, 6, 8, 1, 3, 5, 7, 1, 2, 3, 1, 2, 3]$$

then

$$\leftrightarrow S = [3, 2, 1, 3, 2, 1, 7, 5, 3, 1, 8, 6, 4, 2]$$

See Figures 1 and 2.

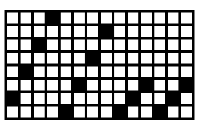

Figure 1. S

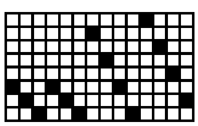

Figure 2. $\leftrightarrow S$

Vertical Reflection

It is also possible to reflect a sequence vertically by reversing the *values*, so that the largest becomes 1, the next-to-largest becomes 2, and so on. If this operation is denoted by $\nu(i)$, then

$$\nu(i) = \gamma(i) - i + 1$$

The operation of vertical reflection is denoted by $\updownarrow S$. For example, if

$$S = \rightarrow[1, 3, 1, 6, 2, 8]$$

then

$$\updownarrow S = \rightarrow[8, 6, 8, 3, 7, 1]$$

See Figures 3 and 4.

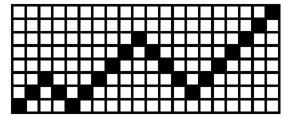

Figure 3. S

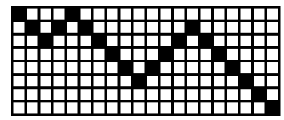

Figure 4. $\updownarrow S$

Palindromes

A palindrome is a sequence that is the same forwards and backwards. A palindrome is created by concatenating a sequence with its horizontal reflection (reversal):

$$S \mid \leftrightarrow S$$

This operation is so important that it has its own notation: $\cap S$. For example, if

$$S = \rightarrow[1, 3, 1, 6, 2, 8]$$

as shown in Figure 3, then

$$\cap S = \rightarrow[1, 3, 1, 6, 2, 8, 2, 6, 1, 3, 1]$$

See Figure 5.

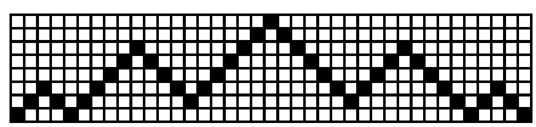

Figure 5. $\cap S$

Note that the duplicate value at the middle is removed, as it is with concatenation [1]. In the case that duplicate removal is not desired,

$$S \mid_{+} \leftrightarrow S$$

can be used.

"Palinforms"

The coined the word "palinforms" refers to concatenations of a sequence with one of its reflections other than the horizontal one.

There are two reflections other than horizontal that can be used to create palinforms: vertical and combined horizontal and vertical. Consider

$$S = \rightarrow[1, 3, 1, 6, 2, 8]$$

Figure 4 above shows the vertical reflection and Figure 6 shows the combined horizontal and vertical reflections.

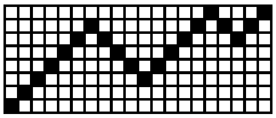

Figure 6. ↕↔*S*

Figures 7 and 8 show the palinforms for these reflections.

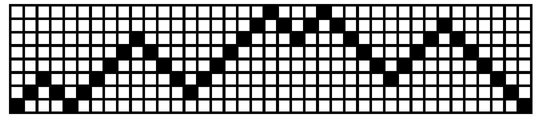

Figure 7. S | ↕*S*

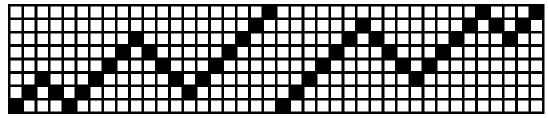

Figure 8. S | ↕↔*S*

Although these palinforms do not have the obvious symmetry of palindromes, the inherent relationships produce visual interest, if of a more subtle form.

Summary

↔*S*	horizontal reflection
↕*S*	vertical reflection
∩*S*	palindrome formation

Reference

1. Ralph E. Griswold, “T-Sequences, Part 2: Extension”, 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_ts02.pdf

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