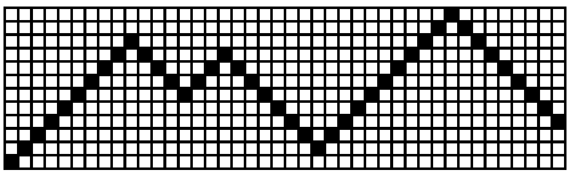
T-Sequences, Part 6: Modular Operations

Modular Reduction

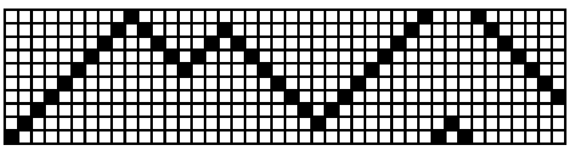
Modular reduction was described in an earlier article as a method of bringing the values in a sequence within a specified range[1]. In terms of t-sequence operations,

$$S \equiv m$$

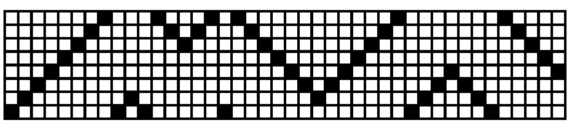
denotes modular reduction of S, shaft-modulo m. See the following figures:



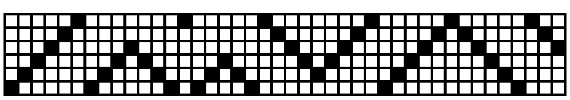
S



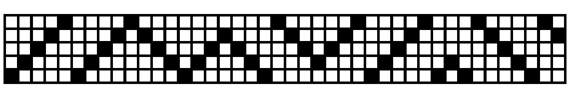
$S \equiv 10$



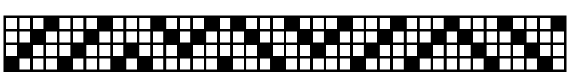
$S \equiv 8$



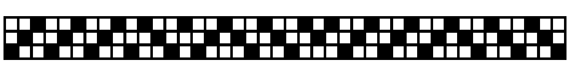
$S \equiv 6$



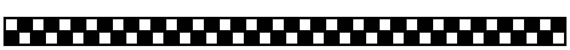
$S \equiv 5$



$S \equiv 4$



$S \equiv 3$

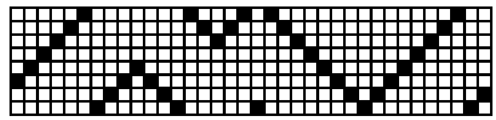


$S \equiv 2$

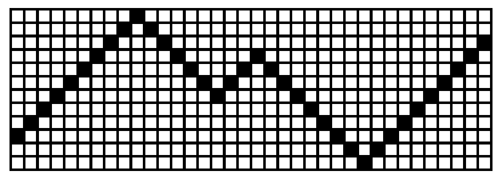
Modular Expansion

Modular expansion, which is the converse of modular reduction, can be used to convert a t-sequence on m shafts to a t-sequence on n shafts, $n \geq m$, in which there is no wrap-around. The result is a sequence whose residues, shaft modulo m, produce the original sequence.

Here is an example:

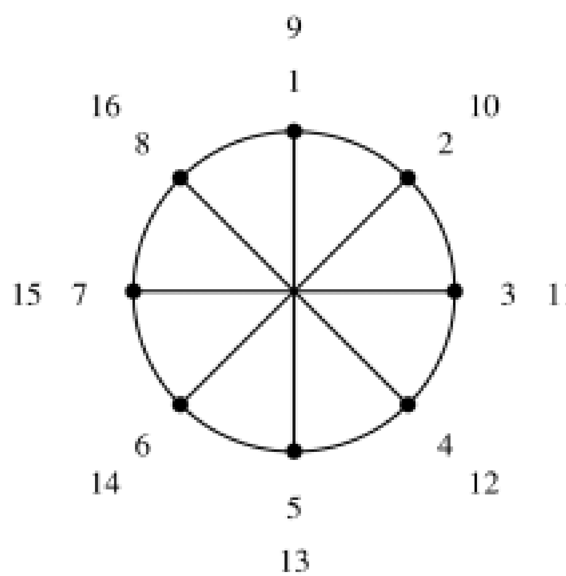


A T-Sequence with Wrap-Around



Wrap-Around Removed by Modular Expansion

The process of modular expansion is simple and relies on the fact that 1 and m are adjacent on the modular wheel. This is illustrated in the modular wheel for 8 shafts:



Starting with $i = 1$, if term $t_i = m$ and $t_{i+1} = 1$, add m to t_{i+1} and all the remaining terms (shifting them upward by m). Similarly, if $t_i =$

1 and $t_{i-1} = m$, subtract m from t_{i-1} and all the remaining terms (shifting them downward by m). Note that adding or subtracting a multiple of m does not affect the residues.

When the process is done, add enough multiples of m to bring the smallest value in the range 1 to m. (The smallest value can be less than 1 but it cannot be greater than m, since t_1 is not greater than m and is not changed by the process.)

The notation

$$\not\equiv S$$

denotes the modular expansion of S.

The relationship between modular reduction and modular expansion is shown by

$$((\not\equiv S) \equiv m) = S$$

Of course, $\not\equiv S$ is not the only sequence whose residues shaft modulo m produces S.

Summary

$S \equiv m$	modular reduction
$\not\equiv S$	modular expansion

Reference

1. Ralph E. Griswold, “Drafting with Sequences”, 2002: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_seqd.pdf

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